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SPEC model selection algorithm for ARCH models: an options pricing evaluation framework

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A number of single ARCH model-based methods of predicting volatility are compared to Degiannakis and Xekalaki's (2005) poly-model standardized prediction error criterion (SPEC) algorithm method in terms of profits from trading actual options of the S&P500 index returns. The results show that traders using the SPEC for deciding which model's forecasts to use at any given point in time achieve the highest profits.

I. Introduction

Degiannakis and Xekalaki (2007) examined the ability of the standardized prediction error criterion (SPEC) model selection algorithm to indicate the ARCH model that generates 'better' volatility predictions with a number of statistical evaluation criteria. In the context of a simulated options market, Xekalaki and Degiannakis (2005) have found that the SPEC algorithm performs 'better' than any other comparative method of model selection in pricing straddles with 1 day to maturity. The present manuscript evaluates the ability of the SPEC algorithm in selecting at each point in time an accurate volatility forecast for the remaining life of a straddle[1] option. The forecasts of option prices are calculated by feeding the volatility estimated by the ARCH models into the Black and Scholes (BS) option pricing model. The obtained results indicate that SPEC has a satisfactory performance in selecting the ARCH models that yield 'better' volatility predictions for option pricing.

II. ARCH Models

For $y_t = \ln(S_t/S_{t-1})$ denoting the continuously compound rate of return from time $t-1$ to t, where S_t is the asset price at time t, a set of ARCH models are estimated. The conditional mean is considered as a κ^{th} order autoregressive process:

$$y_t = c_0 + \sum_{i=1}^{\kappa} (c_i y_{t-i}) + z_t \sigma_t \tag{1}$$

for $z_t \overset{i.i.d.}{\sim} N(0,1)$, and the conditional variance is commonly regarded as one of Assumption (i) a GARCH(p, q) function:

$$\sigma_t^2 = (u_t', \eta_t', w_t')(v, \zeta, \omega)', \tag{2}$$

with $u_t' = (1, \varepsilon_{t-1}^2, \ldots, \varepsilon_{t-q}^2)$, $\eta_t' = 0, w_t' = (\sigma_{t-1}^2, \ldots, \sigma_{t-p}^2)$, $v' = (a_0, \quad a_1, \ldots, a_q)$, $\zeta' = 0$, $\omega' = (b_1, \ldots, b_p)$,

Assumption (ii) an EGARCH(p, q) function:

$$\ln(\sigma_t^2) = (u_t', \eta_t', w_t')(v, \zeta, \omega)', \tag{3}$$

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[1] A straddle option is the purchase of both a call and a put option with the same expiration date and exercise price.

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with $u'_t = (1, |\varepsilon_{t-1}/\sigma_{t-1}|, \ldots, |\varepsilon_{t-q}/\sigma_{t-q}|)$, $\eta'_t = (\varepsilon_{t-1}/\sigma_{t-1}, \ldots, \varepsilon_{t-q}/\sigma_{t-q})$, $w'_t = (\ln(\sigma^2_{t-1}), \ldots, \ln(\sigma^2_{t-p}))$, $v' = (a_0, a_1, \ldots, a_q)$, $\zeta' = (\gamma_1, \ldots, \gamma_q)$, $\omega' = (b_1, \ldots, b_p)$,

Assumption (iii) or as a TARCH(p, q) function:

$$\sigma^2_t = (u'_t, \eta'_t, w'_t)(v, \zeta, \omega)', \tag{4}$$

with $u'_t = (1, \varepsilon^2_{t-1}, \ldots, \varepsilon^2_{t-q})$, $\eta'_t = (d_{t-1}\varepsilon^2_{t-1})$, $w'_t = (\sigma^2_{t-1}, \ldots, \sigma^2_{t-p})$, $v' = (a_0, a_1, \ldots, a_q)$, $\zeta' = (\gamma)$, $\omega' = (b_1, \ldots, b_p)$, $d_t = 1$ if $\varepsilon_t < 0$ and $d_t = 0$ otherwise.

The prediction of the conditional variance at day $t+i$ given the information set available at day t can be computed as:

$\hat{\sigma}^2_{t+i|t} \equiv E(\sigma^2_{t+i}|I_t) = E(u'_{t+i}, \eta'_{t+i}, w'_{t+i}|I_t)(v^{(t)}, \zeta^{(t)}, \omega^{(t)}) = (u'_{t+i|t}, \eta'_{t+i|t}, w'_{t+i|t})(v^{(t)}, \zeta^{(t)}, \omega^{(t)})$. Thus, the AR($\kappa$)GARCH($p,q$), AR($\kappa$)EGARCH($p,q$) and AR($\kappa$)TARCH($p,q$) models are applied, for $\kappa = 0, \ldots, 4$, $p = 0, 1, 2$ and $q = 1, 2$.

III. The SPEC Model Selection Algorithm

Assume that a set of M candidate ARCH models is available and that the 'most suitable' model is sought for predicting conditional volatility. The ARCH model, with the lowest value of the sum of the T most recent estimated squared standardized one-step-ahead prediction errors, $\sum_{t-1}^{T} \hat{\varepsilon}^2_{t+1|t}/\hat{\sigma}^2_{t+1|t}$, can be considered for obtaining one-step-ahead forecasts of the conditional volatility. Assume further that the M competing ARCH processes have been estimated using a rolling sample of n observations. The SPEC algorithm for selecting the 'most suitable' of the M candidate models at each of a series of points in time is comprised of the following steps.

For model m, ($m = 1, 2, \ldots, M$) and for each point in time t, ($t = n, n+1, \ldots$), the vector of coefficients $\hat{\theta}^{(m)(t)} \equiv (\hat{\beta}^{(m)(t)}, \hat{v}^{(m)(t)}, \hat{\zeta}^{(m)(t)}, \hat{\omega}^{(m)(t)})$ is estimated using a rolling sample of n observations. Using the vector of coefficients $\hat{\theta}^{(m)(t)}$, compute $R^{(m)}_{T+n} \equiv \sum_{t=n}^{T+n+1} \hat{\varepsilon}^{2(m)}_{t+1|t}/\hat{\sigma}^{2(m)}_{t+1|t}$

The 'most suitable' model to forecast volatility at time $T+n$ is the model m with the minimum value of $R^{(m)}_{T+n}$. The algorithm is repeated for each of a sequence of points in time for the selection of the 'most appropriate' model to be used for obtaining a volatility forecast for the next point in time.

IV. Measuring the Forecasting Performance

The BS formula to price call and put options at day $t+1$ given the information available at day t, with τ days to maturity, denoted, respectively, by $C^{(\tau)}_{t+1|t}$ and $P^{(\tau)}_{t+1|t}$, can be presented in the following form:

$$C^{(\tau)}_{t+1|t} = S_t e^{-\gamma_t \tau} N(d_1) - K e^{-rf_t \tau} N(d_2) \tag{5}$$

$$P^{(\tau)}_{t+1|t} = -S_t e^{-\gamma_t \tau} N(-d_1) + K e^{-rf_t \tau} N(-d_2)$$

$$d_1 = \frac{\ln(S_t/K) + \left(rf_t - \gamma_t + 1/2\left(\sigma^{(\tau)}_{t+1|t}\right)^2\right)\tau}{\sigma^{(\tau)}_{t+1|t}\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma^{(\tau)}_{t+1|t}\sqrt{\tau}, \tag{6}$$

where, S_t is the daily closing stock price as a forecast of S_{t+1}, rf_t is the daily risk free interest rate, γ_t is the daily dividend yield, K is the exercise price, $N(.)$ is the cumulative normal distribution function and $\sigma^{(\tau)}_{t+i|t} = \sqrt{\tau^{-1}\sum_{i=2}^{\tau+1}\hat{\sigma}^2_{t+1|t}}$ is the volatility during the life of the option.

If the straddle price forecast is greater than the market straddle price, the straddle is bought. If the straddle price forecast is less than the market straddle price, the straddle is sold:

$$\text{If } C^{(\tau)}_{t+1|t} + P^{(\tau)}_{t+1|t} > P^{(\tau)}_t + C^{(\tau)}_t \Rightarrow \text{The straddle is bought at time } t \tag{7}$$

$$\text{If } C^{(\tau)}_{t+1|t} + P^{(\tau)}_{t+1|t} < P^{(\tau)}_t + C^{(\tau)}_t \Rightarrow \text{The straddle is sold at time } t \tag{8}$$

The rate of return from straddle trading is:

$$NRT_t = \begin{cases} \dfrac{C_t + P_t - C_{t-1} - P_{t-1} - X}{C_{t-1} + P_{t-1}}, & \text{if } C^{(\tau)}_{t|t-1} + P^{(\tau)}_{t|t-1} - C_{t-1} - P_{t-1} > F \\ \dfrac{C_{t-1} + P_{t-1} - C_t - P_t - X}{C_{t-1} + P_{t-1}} + r_{f_t}, & \text{if } C_{t-1} + P_{t-1} - C^{(\tau)}_{t|t-1} - P^{(\tau)}_{t|t-1} > F \\ rf_t, & \text{otherwise,} \end{cases} \tag{9}$$

where X denotes the transaction cost. We assume that the straddles are traded only when the absolute difference between the forecast and the actual straddle price exceeds the amount of the filter, F. Otherwise, agents are assumed to invest at the risk free rate.

V. Datasets

The data set consists of 1064 S & P500 stock index daily returns in the period from 14 March 1996 to 2 June 2000. A rolling sample of constant size equal to $n = 500$ is considered. Hence, the first one-step-ahead volatility prediction, $\hat{\sigma}^2_{t+1|t}$, is available at time $t = 500$, or on 11 March 1998. The use of a restricted sample size incorporates changes in the trading behaviour more efficiently.[2]

The S & P500 index options data were obtained from the Datastream for the period from 11 March 1998 through 2 June 2000, totally 564 trading days. Proper data are available for 456 trading days. In order to minimize the biasedness of the BS formula, only the straddle options with exercise prices closest to the index level, maturity longer than 10 trading days and trading volume >100 were considered. Practice has shown that the BS pricing model tends to misprice deep-out-of-the-money and deep-in-the-money options, while it works better for near-the-money options (see, e.g. Daigler, 1994, p. 153). Also, a maturity period of length no shorter than 10 trading days is considered to avoid mispricings attributable to causes of practical as well as of theoretical nature.

VI. Results

The day-by-day rates of return are reflective of the corresponding predictive performances of the models. We have on the one hand traders who always choose to use one and the same ARCH model for their forecasts and traders who at each point in time choose to use the ARCH model suggested by the SPEC algorithm on the other.

There are 85 traders and each trader employs an ARCH model to forecast future volatility and straddle prices. For each trader, the daily rate of return from trading straddles for 456 days is computed according to Equation 9.[3] A transaction cost of \$2 that reflects the bid – ask spread is considered. Various values for the filter F are applied, i.e. \$0, \$1.25, \$1.75, \$2.00, \$2.25, \$2.75, \$3.50. For $F = \$3.50$, the trader using the AR(3)GARCH(0,2) forecasts makes the highest daily profit of 1.35% with a corresponding SD of 15.24% and a t-ratio of 1.89 (or p-value 0.06).

Applying the SPEC model selection algorithm, the sum of squared standardized one-step-ahead prediction errors, $\sum_{t=1}^{T} \hat{z}^2_{t|t-1}$, was estimated considering various values for T, and, in particular, $T = 5(5)80$.[4] Thus, it is assumed that there are 16 traders each of which uses on each trading day, the ARCH model picked by the SPEC algorithm to forecast volatility and straddle prices for the next trading day. With a filter of \$3.5, the trader utilizing the SPEC algorithm with $T = 5$ achieves the highest profit of 1.46% per day with a corresponding SD of 15.85% and a t-ratio of 1.97 (or p-value 0.05). Even marginally, the SPEC(5) model selection algorithm generates higher returns than those achieved by any other trader using only a single ARCH model.[5] Thus, the SPEC model selection algorithm appears to have a satisfactory performance in selecting those models that generate 'better' volatility predictions.

One might take the view that the SPEC algorithm would favour the model that produces higher volatility forecasts. However, comparing the SPEC algorithm with a model selection algorithm that was constructed so as to select the model with the maximum sum of the T most recent estimated one-step-ahead volatility forecasts (denoted by MAXVAR) for various values of T revealed that this is not the case. In none of the cases did the daily profits achieved by traders using MAXVAR(T) exceed the profits made by traders using SPEC(T) for $T = 5(5)80$. Only in an average of 5% of the trading days did the MAXVAR(T) algorithm pick the same models as those picked by the SPEC(T) algorithm.

Considering the squared daily returns as a proxy for the unobserved actual variance, a set of statistical criteria to measure the closeness of the forecasts to the realizations are also estimated:

Squared Error of Conditional Variance (SEVar):

$$\sum_{t=1}^{T} \left(\left(\hat{\sigma}^2_{t+1|t} - y^2_{t+1} \right)^2 \right) \tag{10}$$

Absolute Error of Conditional Variance (AEVar):

$$\sum_{t=1}^{T} \left(\left| \hat{\sigma}^2_{t+1|t} - y^2_{t+1} \right| \right) \tag{11}$$

[2] See for example Xekalaki and Degiannakis (2005).
[3] Because of the large amount of data, tables with all the ARCH models are available upon request.
[4] $T = a(b)c$ denotes $T = a, a+b, a+2b, \ldots, c-b, c$.
[5] For any value for the filter, the SPEC algorithm generates the highest returns, but the p-value is the lowest for $F = \$3.5$. The Sharpe ratios, which are available upon request, were also calculated giving similar results.

Table 1. The net rate of return, computed as in Equation 9, from trading straddles on the S & P500 index based on the SPEC algorithm and the model selection algorithms presented in Equations 10–18, with $2.00 transaction costs and a $3.5 filter

Model selection method	Sample size	Mean	*t*-ratio
SPEC	T = 5	1.46%	1.97
SEVar	T = 40	0.61%	0.80
AEVar	T = 60	0.76%	1.03
SEDev	T = 60	0.74%	0.97
AEDev	T = 60	0.81%	1.08
HASEVar	T = 10	1.10%	1.47
HAAEVar	T = 40	1.24%	1.65
HASEDev	T = 20	0.90%	1.18
HAAEDev	T = 30	1.12%	1.45
LEVar	T = 80	0.75%	1.00

Notes: The column titled sample size refers to the sample size, *T*, for which the corresponding model selection algorithm leads to the highest rate of return.

Squared Error of Conditional SD (SE Dev):

$$\sum_{t=1}^{T}\left(\left(\hat{\sigma}_{t+1|t}-|y_{t+1}|\right)^2\right) \tag{12}$$

Absolute Error of Conditional SD (AEDev):

$$\sum_{t=1}^{T}\left(\left|\hat{\sigma}_{t+1|t}-|y_{t+1}|\right|\right) \tag{13}$$

Heteroscedasticity Adjusted Squared Error of Cond. Variance (HASEVar):

$$\sum_{t=1}^{T}\left(1-\left(\frac{y_{t+1}^2}{\hat{\sigma}_{t+1|t}^2}\right)\right)^2 \tag{14}$$

Heteroscedasticity Adjusted Absolute Error of Cond. Variance (HAAEVar):

$$\sum_{t=1}^{T}\left(1-\left|\frac{y_{t+1}^2}{\hat{\sigma}_{t+1|t}^2}\right|\right) \tag{15}$$

Heteroscedasticity Adjusted Squared Error of Cond. St. Deviation (HASEDev):

$$\sum_{t=1}^{T}\left(1-\left(\frac{|y_{t+1}|}{\hat{\sigma}_{t+1|t}}\right)\right)^2 \tag{16}$$

Heteroscedasticity Adjusted Absolute Error of Cond. St. Deviation (HAAEDev):

$$\sum_{t=1}^{T}\left|1-\left(\frac{|y_{t+1}|}{\hat{\sigma}_{t+1|t}}\right)\right| \tag{17}$$

Logarithmic Error of Conditional Variance (LEVar):

$$\sum_{t=1}^{T}\left(\ln\left(\frac{y_{t+1}^2}{\hat{\sigma}_{t+1|t}^2}\right)^2\right) \tag{18}$$

Applying the SPEC model selection algorithm, the sum of squared standardized one-step-ahead prediction errors, $\sum_{t=1}^{T}\hat{\varepsilon}_{t+1|t}^2/\hat{\sigma}_{t+1|t}^2$, was estimated considering various values for T. Therefore, each of the model selection criteria is computed considering various values for T, and, in particular, $T = 10(10)80$. Selecting a strategy based on any of several competing methods of model selection naturally amounts to selecting the ARCH model that, at each of a sequence of points in time, has the lowest value of the evaluation function.

In none of the cases, did the daily returns come out to be higher than the returns achieved by the SPEC algorithm. Table 1 presents the daily rate of returns based on the ARCH models selected by the 10-model selection methods.[6] The HAAEVar selection algorithm, for $T = 40$, yielded the highest daily profit (1.24%) with a t-ratio of 1.65.

VII. Conclusion and Suggestions for Further Research

The results of our study showed that the SPEC algorithm outperformed all of the single ARCH model-based methods as well as a set of other methods of model selection. This is in agreement with Xekalaki and Degiannakis's (2005) findings

[6] Detailed tables for the daily rate of return from trading straddles based on the ARCH models selected by the 10-model selection methods are available upon request.

from a comparative study of ARCH model selection algorithms performed on the basis of simulated options data, who also showed that the SPEC algorithm for $T = 5$ achieved the highest rate of return.

The validity of the variance forecasts depends on which option pricing formula is used. Even if one could find the model, which predicts the volatility precisely, it is well known that the BS formula does not describe the dynamics of pricing the options perfectly. In future research, the estimation of ARCH-based option pricing models such that of Duan (1995) and Heston and Nandi (2000) is suggested.

The SPEC algorithm does increase the volatility prediction accuracy and can be considered as a tool in picking the model that would yield the best volatility prediction. However, the SPEC algorithm provides profits significantly greater than 0 under a perfect framework of no commissions. Only the bid-ask spread was taken into account.[7] Under realistic transaction charges for a trader and market impact costs, the daily profits are wiped out. If someone could really gain 1.46% per trading day after commissions, the presented results would make a good case for market inefficiency or at least for a huge temporary inefficiency.

References

Daigler, R. T. (1994) *Advanced Option Trading*, Probus Publishing Company, Chicago, USA.

Degiannakis, S. and Xekalaki, E. (2005) Predictability and model selection in the context of ARCH models, *Journal of Applied Stochastic Models in Business and Industry*, **21**, 55–82.

Degiannakis, S. and Xekalaki, E. (2007) Assessing the performance of a prediction error criterion model selection algorithm in the context of ARCH Models, *Applied Financial Economics*, **17**, 149–71.

Duan, J. (1995) . The GARCH option pricing model, *Mathematical Finance*, **5**, 31–2.

Heston, S. L. and Nandi, S. (2000) A closed-form GARCH option valuation model, *The Review of Financial Studies*, **13**, 585–625.

Xekalaki, E. and Degiannakis, S. (2005) Evaluating volatility forecasts in option pricing in the context of a simulated options market, *Computational Statistics and Data Analysis*, **49**, 611–29.

[7] On average, a transaction cost of 2% for each option contract was considered, or 8% ($2\% \times 4$) for trading straddles. However, the bid-ask spread could be even wider. A straddle trader might stipulate limit prices for both put and call options to narrow it down, but, in most cases, the orders would remain unfilled.