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RESEARCH ARTICLE

On the relationship between process capability indices and the proportion of conformance

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ABSTRACT

The proportion of conformance of a process, i.e. the probability of producing within the specification area, is related to the majority of the most commonly used process capability indices (PCIs). In this article, the relationship of the four most widely used indices, i.e. C_p, C_{pk}, C_{pm} and C_{pmk} to the proportion of conformance is examined in the case of normal processes. Results known in the literature are presented along with several new ones. Various plots, revealing interesting aspects of these relationships are also provided.

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Normal process; process capability indices; proportion of conformance; specification area; statistical process control

1. Introduction

Process capability indices (PCIs) have been developed in order to measure the capability of a process to produce according to some given specifications. These specifications are associated with a process characteristic (X) and include the lower specification limit (L), the upper specification limit (U) and the target value (T). The values of L and U correspond to the minimum and the maximum acceptable process value, respectively, and can be used for the definition of the specification area. Actually, the specification area of a process is defined as the interval that includes all the acceptable process values, i.e. the interval $(L,\ U)$.

A review of the properties and the suggested techniques for the estimation of a large number of PCIs is provided by Kotz and Johnson (1993, 2002), Kotz and Lovelace (1998), Pearn and Kotz (2006) and Wu, Pearn and Kotz (2009). Furthermore, Spiring, Leung, Cheng and Yeung (2003) and Yum and Kim (2011) provide an extensive coverage of the bibliography on PCIs. Among the plethora of the suggested PCIs, the most widely used are C_p, C_{pk}, C_{pm} and C_{pmk}. These four indices, which can be considered as special cases of the general index $C_p\ (\mu,\ v)$ introduced by Vannman (1995), have been developed for processes described through a characteristic whose values are normally distributed. Therefore, in the current article, we assume that the quality characteristic of the studied process follows a normal distribution.

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The proportion of conformance (yield) of a process is defined as the probability of producing within the specification area and plays undoubtedly an important role in the measurement of process capability, whereas the values of the majority of the suggested PCIs – including the four indices mentioned before – are related to it. Besides, Carr (1991) suggested the direct use of its value as a measure of the capability of a process, while Yeh and Bhattacharya (1998) and Perakis and Xekalaki (2002, 2005) suggested the use of some indices that are functions of it.

Under the assumption of a normally distributed process, the proportion of conformance is given by

$$p = P(L<X<U) = \Phi\left(\frac{U-\mu}{\sigma}\right) - \Phi\left(\frac{L-\mu}{\sigma}\right), \tag{1}$$

where μ, σ are the mean and the standard deviation of the process, respectively and $\Phi(.)$ denotes the cumulative distribution function of the standard normal distribution.

Establishing relationships between capability indices and the yield of a process has attracted interest and various results have appeared in the literature, e.g. Boyles (1991), Kotz and Johnson (1993), Johnson and Kotz (1995) and Kotz and Lovelace (1998). In this article, the relationship of the four basic PCIs to the proportion of conformance is thoroughly investigated in the case of normal processes and several formulae that connect their values both known in the literature as well as new ones are provided (Sections 2 through to 5). These are accompanied by several graphs elucidating various aspects of these relationships. A formula that expresses the proportion of conformance as a function of C_p, C_{pk}, C_{pm} and C_{pmk} is obtained in Section 6 and a discussion of the findings is provided in Section 7.

2. The index C_p

The index C_p, initially suggested by Juran, Gryna, and Bingham (1974), is the simplest and most widely used capability index. It is defined as the ratio of the allowable process spread (determined by the length of the specification area) to the actual process spread (associated with the value of σ), thus

$$C_p = \frac{U-L}{6\sigma}.$$

The value of C_p for a specific process does not contain enough information for obtaining its proportion of conformance. Nonetheless, it can be used for obtaining an upper bound that the actual value of p cannot exceed. This bound is determined through the inequality

$$p \leq 2\Phi(3C_p) - 1, \tag{2}$$

(see, e.g. Kotz and Johnson (1993)). The equality in inequality (2) is satisfied only in the case where the process mean coincides with the mid-point of the specification area, given by $M = (L+U)/2$. Actually, if $\mu = M$, then the values of $U-\mu$ and $L-\mu$ that appear in the numerators of the arguments in Equation (1) are equal to d and $-d$, respectively, where d denotes the half length of the specification area, i.e. $d = (U-L)/2$. Therefore, the right hand side of Equation (1) simplifies to $\Phi(d/\sigma) - \Phi(-d/\sigma)$. Taking into account the fact that the index C_p can be expressed in

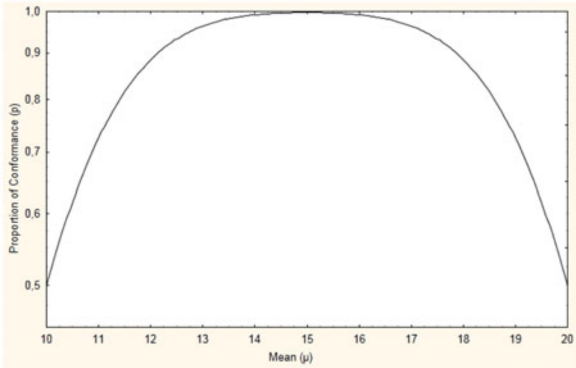

Figure 1. Plot of the proportion of conformance for various values of μ assuming that $C_p = 1$, $L = 10$ and $U = 20$.

terms of d in the form

$$C_p = \frac{d}{3\sigma},$$

it follows that the proportion of conformance of a process with $\mu = M$ is equal to $\Phi(3C_p) - \Phi(-3C_p)$ or, equivalently, to $2\Phi(3C_p) - 1$. Due to the symmetric form of the normal distribution, the value of the proportion of conformance is always maximized for a fixed value of σ or, equivalently, for a fixed value of C_p (as the value of C_p depends solely on σ) when $\mu = M$. Therefore, it cannot exceed $2\Phi(3C_p) - 1$ for any process mean different from M and, thus, inequality (2) is always satisfied.

The result given in inequality (2) is depicted in Figure 1, which is a plot of p for various values of μ, assuming, without loss of generality, that $C_p = 1$, $L = 10$ and $U = 20$. The value of σ corresponding to these values of C_p, L and U is given by 1.667. One may observe from this figure that the proportion of conformance is maximized (taking the value $2\Phi(3) - 1 = 0.9973$) when the process mean is equal to the mid-point $M = 15$ of the specification area and decreases symmetrically as μ moves towards the specification limits.

3. The index C_{pk}

The index C_{pk} was suggested by Kane (1986) in order to provide a more effective measure of the capability of a process than C_p. Actually, this index takes into account both process parameters (μ, σ) and is defined as

$$C_{pk} = \min\left\{\frac{\mu - L}{3\sigma}, \frac{U - \mu}{3\sigma}\right\}.$$

The value of C_{pk} is also not sufficient for assessing the proportion of conformance of a process, but it is more informative than that of C_p, while it makes possible the calculation of two limits within which the yield of a process always lies. In particular, as Boyles (1991) points out, the yield of a process lies always between the values

$$2\Phi(3C_{pk}) - 1 \text{ and } \Phi(3C_{pk}). \tag{3}$$

Indeed, consider a process with a known value of C_{pk} whose mean lies within the first half of the specification area, i.e. within the interval $[L, M]$. In this case, the process mean is closer to the lower specification limit and thus the value of C_{pk} equals

$$C_{pk} = \frac{\mu - L}{3\sigma}.$$

In the light of this result, we deduce that the right hand side term of Equation (1) can be written as

$$\Phi\left(\frac{U-\mu}{\sigma}\right) - \Phi(-3C_{pk}) = \Phi(3C_{pk}) - 1 + \Phi\left(\frac{U-\mu}{\sigma}\right). \tag{4}$$

From the definition of C_{pk} and since $L \le \mu \le M$, it follows that $3C_{pk} \le (U-\mu)/\sigma$ and, hence, the proportion of conformance cannot be smaller than $2\Phi(3C_{pk})-1$, whereas $\Phi(.)$ is by definition a non-decreasing function. On the other hand, the value of p cannot exceed $\Phi(3C_{pk})$, because the largest value that the term $\Phi((U-\mu)/\sigma)$ in Equation (4) may take is 1, leading to a yield equal to $\Phi(3C_{pk})$. Similar conclusions may be drawn on the assumption that the mean of the process lies within the interval $[M, U]$. Hence, the value of p cannot lie outside of the bounds given in (3).

Despite the fact that knowledge of the value of either C_p or C_{pk} is not sufficient for obtaining the value of the proportion of conformance of a process, it may be easily proved that knowledge of the values of both of these indices allows one to obtain the value of the proportion of conformance. Indeed, the proportion of conformance of a process can be expressed as (Kotz & Johnson, 1993)

$$p = \Phi\big(3\big(2C_p - C_{pk}\big)\big) - \Phi\big(-3C_{pk}\big). \tag{5}$$

To show this, assume, without loss of generality, that $L \le \mu \le M$. Then, the value of the index C_{pk} is given by

$$C_{pk} = \frac{\mu - L}{3\sigma}.$$

Moreover,

$$2C_p - C_{pk} = \frac{U-L}{3\sigma} - \frac{\mu - L}{3\sigma} = \frac{U-\mu}{3\sigma}.$$

Therefore, the proportion of conformance equals

$$p = \Phi\left(\frac{U-\mu}{\sigma}\right) - \Phi\left(\frac{L-\mu}{\sigma}\right) = \Phi(3(2C_p - C_{pk})) - \Phi(-3C_{pk}).$$

A similar result is obtained if the mean is assumed to be located at the upper half of the specification area. This establishes Equation (5).

4. The index C_{pm}

The index C_{pm}, was initially suggested by Hsiang and Taguchi (1985), but its use became widespread after the publication of the articles by Chan, Cheng and Spiring (1988) and Boyles (1991), who made a thorough study of its properties and the problem of its estimation. The index C_{pm} is defined as

$$C_{pm} = \frac{U - L}{6\sqrt{\sigma^2 + (\mu - T)^2}}.$$

In contrast to the two previous indices, C_{pm} takes into account, except for the specification limits, the target value T of a process as well.

Johnson and Kotz (1995) examined the relationship between the value of C_{pm} and the expected proportion of non-conforming items. Since the yield of a process is defined as the complement of the expected proportion of non-conforming items, their results are presented in the sequel, in terms of the proportion of conformance so as to be comparable to those for the indices C_p and C_{pk}, given above. For simplicity, Johnson and Kotz (1995) assumed that $-L = U = h$ and $T = 0$ and concluded that the proportion of conformance of a process can be written in the form

$$p = \Phi\left(\frac{h - \mu}{\sqrt{\lambda^2 - \mu^2}}\right) - \Phi\left(\frac{-h - \mu}{\sqrt{\lambda^2 - \mu^2}}\right), \tag{6}$$

where $\lambda = h/(3C_{pm})$. Moreover, they investigated the properties of Equation (6). The basic findings (see also Kotz and Lovelace (1998)) about this equation were that it is symmetric about 0, it has a (local) minimum at $\mu = 0$ if $C_{pm} > 1/\sqrt{3}$ and it has a (local) maximum at $\mu = 0$ if $C_{pm} < 1/\sqrt{3}$. Johnson and Kotz (1995) also noted that Equation (6) decreases with $|\mu|$ for all μ if $C_{pm} < 1/3$ and has at least a local minimum at $\mu \pm \mu_0$ for some $\mu_0 \neq 0$ if $1/3 < C_{pm} < 1/\sqrt{3}$. According to Kotz and Lovelace (1998), the most interesting property of Equation (6) is its minimization at $\mu = 0$ if $C_{pm} > 1/\sqrt{3}$, since this property shows that, for relatively large values of C_{pm}, in particular for values greater than $1/\sqrt{3}=$ 0.577, the proportion of conformance is minimized when the process mean coincides with the target value.

In the sequel, a generalization of Equation (6) is given that connects the proportion of conformance to the value of C_{pm} for any values of L, U and T. In particular, it can be shown that

$$p = \Phi\left(\frac{U - \mu}{\sqrt{\left(\frac{d}{3C_{pm}}\right)^2 - (\mu - T)^2}}\right) - \Phi\left(\frac{L - \mu}{\sqrt{\left(\frac{d}{3C_{pm}}\right)^2 - (\mu - T)^2}}\right). \tag{7}$$

To prove this, consider that

$$C_{pm}^2 = \frac{d^2}{9\sigma^2 + 9(\mu - T)^2}. \tag{8}$$

Solving Equation (8) for σ, we obtain

$$\sigma = \sqrt{\left(\frac{d}{3C_{pm}}\right)^2 - (\mu - M)^2}, \tag{9}$$

which combined with Equation (1) leads to Equation (7). If the specifications of the process are symmetric, i.e. if $T = M$, Equation (7) becomes

$$p = \Phi\left(\frac{U-\mu}{\sqrt{\left(\frac{d}{3C_{pm}}\right)^2 - (\mu - M)^2}}\right) - \Phi\left(\frac{L-\mu}{\sqrt{\left(\frac{d}{3C_{pm}}\right)^2 - (\mu - M)^2}}\right). \quad (10)$$

Moreover, if the mean of the process coincides with the mid-point of the specification area, Equation (10) simplifies to

$$p = \Phi\left(\frac{U-M}{\frac{d}{3C_{pm}}}\right) - \Phi\left(\frac{L-M}{\frac{d}{3C_{pm}}}\right) = \Phi(3C_{pm}) - \Phi(-3C_{pm}) = 2\Phi(3C_{pm}) - 1.$$

As pointed out by Kotz and Lovelace (1998), the value $2\Phi(3C_{pm})-1$ determines a lower bound for the proportion of conformance for given value of C_{pm}, provided that the value of C_{pm} is sufficiently large.

A better understanding of the form of Equation (7) can be attained via Figures 2 and 3, which are contour plots of this equation in the $(\mu,\ C_{pm})$ plane, under the assumption that $L = 10$, $U = 20$ and $T = M = 15$. More specifically, the two plots indicate the combinations of μ and C_{pm} that lead to the same value of p, for six different choices of p. The first of these plots corresponds to large values of C_{pm}, while the second corresponds to small ones. The reason why we provide separate plots for large and small values of C_{pm} is the fact that the behaviour of the examined equation changes substantially as the value of C_{pm} changes (Recall also the properties of Equation (6) noted by Johnson and Kotz (1995)).

Inspecting both plots, one may observe that keeping the value of the process mean fixed the value of p increases, as one would expect, as the value of C_{pm} increases. On the other hand, if one keeps the value of C_{pm} fixed, then the value of p increases when μ becomes closer to T only if the value of C_{pm} is very small (Figure 3). On the contrary, for large values of C_{pm}, the proportion of conformance increases as the mean moves further and further away from the target value to the direction of any of the specification limits (Figure 2).

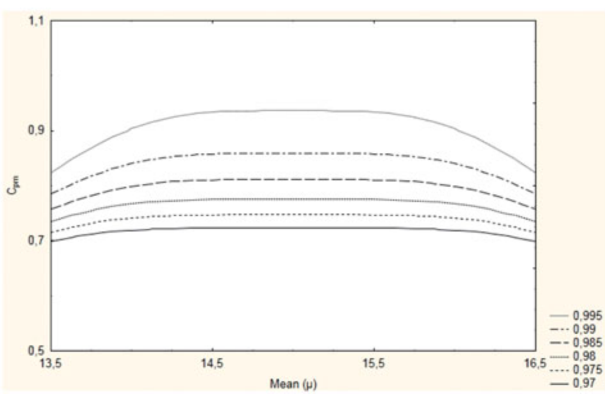

Figure 2. Contour plot of Equation (7) in the (μ, C_{pm}) plane for large values of C_{pm} and specifications given by $L = 10$, $U = 20$ and $T = M = 15$.

Finally, it is interesting to note that the two plots – and particularly that in Figure 2 corresponding to large values of C_{pm} – do not cover a very wide range of values of μ and C_{pm}. The reason for this lies in the fact that, through the definition of C_{pm}, a bound is imposed on the distance between μ and T, i.e.

$$|\mu - T| < \frac{U - L}{6C_{pm}}.$$

Hence, it is not possible to have a process with mean far from the target but with a large C_{pm}.

Revealing of the behaviour of Equation (7) are also Figures 4–7, which are plots of Equation (7) as a function of the process mean for some fixed values of C_{pm}. In particular, Figure 4 corresponds to $C_{pm} = 1.5$, Figure 5 corresponds to $C_{pm} = 1$, Figure 6 corresponds to $C_{pm} = 0.5$ and finally, Figure 7 corresponds to $C_{pm} = 0.3$.

From these plots, one may observe that for relatively large values of C_{pm} (Figures 4 and 5) the proportion of conformance is minimized if $\mu = T$ and increases symmetrically as μ moves towards the specification limits, while, for very small values of C_{pm},

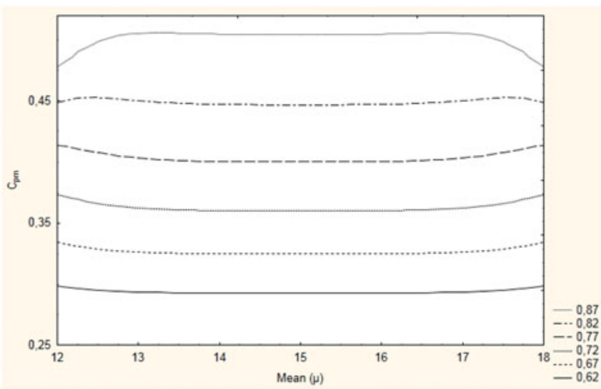

Figure 3. Contour plot of Equation (7) in the (μ, C_{pm}) plane for small values of C_{pm} and specifications given by $L = 10,\ U = 20$ and $T = M = 15$.

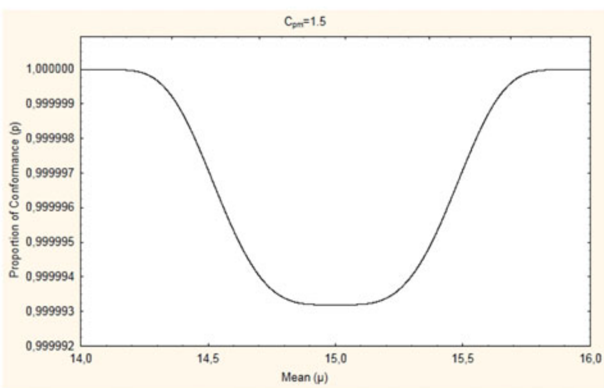

Figure 4. Plot of Equation (7) for various values of μ assuming that $C_{pm} = 1.5$, $L = 10,\ U = 20$ and $T = M = 15$.

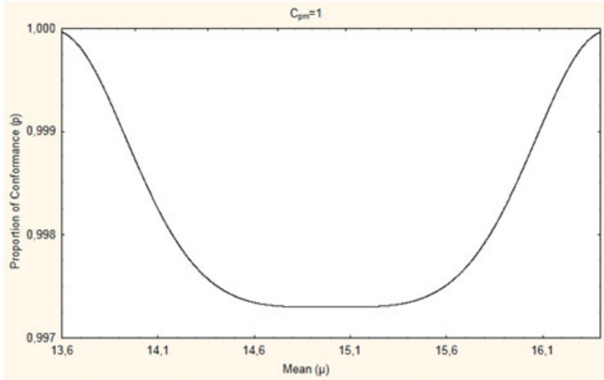

Figure 5. Plot of Equation (7) for various values of μ assuming that $C_{pm} = 1$, $L = 10, U = 20$ and $T = M = 15$.

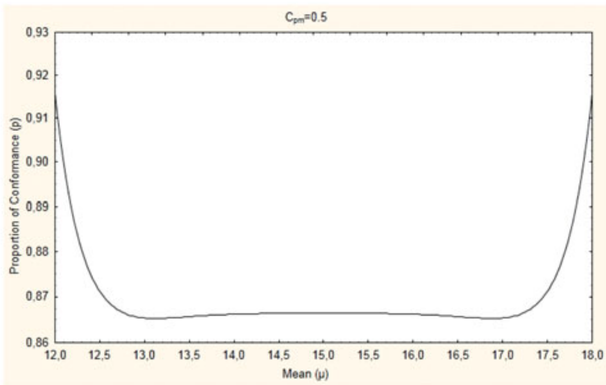

Figure 6. Plot of Equation (7) for various values of μ assuming that $C_{pm} = 0.5$, $L = 10, U = 20$ and $T = M = 15$.

the proportion of conformance is maximized when $\mu = T$ and decreases symmetrically as μ moves towards the specification limits (Figure 7). Finally, when $C_{pm} = 0.5$ (Figure 6) Equation (7) is minimized at two values that are equidistant from T. This property is in agreement with the property of Equation (6) shown by Johnson and Kotz (1995) according to which Equation (6) has at least a local minimum at $\mu \pm \mu_0$ for some $\mu_0 \neq 0$ if $1/3 < C_{pm} < 1/\sqrt{3}$. In Figures 4–7 the specifications are assumed to be $L = 10,\ U = 20$ and $T = M = 15$.

5. The index C_{pmk}

The most elaborate among the four basic capability indices is C_{pmk}. This index was proposed initially by Choi and Owen (1990) with the name C_{pn}. However, its properties were re-examined in more detail by Pearn, Kotz and Johnson (1992), who used the name C_{pmk}, because this index comprises of the numerator of C_{pk} and the denominator of C_{pm}. Since then, the notation C_{pmk} has been adopted for this index by almost all the authors who have studied the properties of this index, such as Wright (1998) and Pearn, Yang, Chen and Lin (2001).

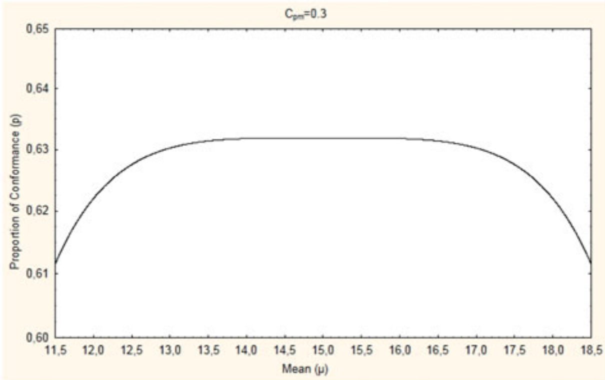

Figure 7. Plot of Equation (7) for various values of μ assuming that $C_{pm} = 0.3$, $L = 10, U = 20$ and $T = M = 15$.

The index C_{pmk} is defined as

$$C_{pmk} = \min\left\{\frac{\mu - L}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{U - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\}.$$

No exact relationship between the proportion of conformance and the index C_{pmk} seems to exist in the literature. As shown in the sequel, if $L \le \mu \le M$, the proportion of conformance can be written in terms of C_{pmk} as

$$p = \Phi\left(\frac{U - \mu}{\sqrt{\left(\frac{\mu - L}{3C_{pmk}}\right)^2 - (\mu - T)^2}}\right) - \Phi\left(\frac{L - \mu}{\sqrt{\left(\frac{\mu - L}{3C_{pmk}}\right)^2 - (\mu - T)^2}}\right). \tag{11}$$

The corresponding expression for the case that $M \le \mu \le U$ is

$$p = \Phi\left(\frac{U - \mu}{\sqrt{\left(\frac{U - \mu}{3C_{pmk}}\right)^2 - (\mu - T)^2}}\right) - \Phi\left(\frac{L - \mu}{\sqrt{\left(\frac{U - \mu}{3C_{pmk}}\right)^2 - (\mu - T)^2}}\right). \tag{12}$$

Equation (11) can be obtained by considering that if $L \le \mu \le M$, then the value of the index C_{pmk} is given by

$$C_{pmk} = \frac{\mu - L}{3\sqrt{\sigma^2 + (\mu - T)^2}}. \tag{13}$$

Solving Equation (13) for σ one obtains

$$\sigma = \sqrt{\left(\frac{\mu - L}{3C_{pmk}}\right)^2 - (\mu - T)^2}, \tag{14}$$

which, combined with Equation (1), leads to Equation (11). Similarly, one may verify the validity of Equation (12). Moreover, it can be verified easily that if $\mu = M = T$, both Equations (11) and (12) simplify to

$$p = 2\Phi(3C_{pmk}) - 1. \tag{15}$$

The value of p given in the right-hand side of Equation (15) is the lowest possible value of the proportion of conformance for a process with known value of C_{pmk}, while, as shown graphically in the sequel, the value of p is minimized for a fixed value of C_{pmk} if $\mu = M = T$.

Let us now give some plots that clarify the relationship of C_{pmk} and p. In particular, in the sequel, contour plots of Equations (11) and (12) in the $(\mu,\ C_{pmk})$ plane are provided. As in the case of C_{pm}, separate plots are provided for small and large index values. More specifically, Figures 8 and 9 correspond to Equation (11) for large and small values of C_{pmk}, respectively, whereas Figures 10 and 11 correspond to Equation (12) for large and small values of C_{pmk}, respectively.

From Figures 8–11 one may observe that, keeping the value of C_{pmk} constant, the proportion of conformance is maximized as μ moves far from the target value, for

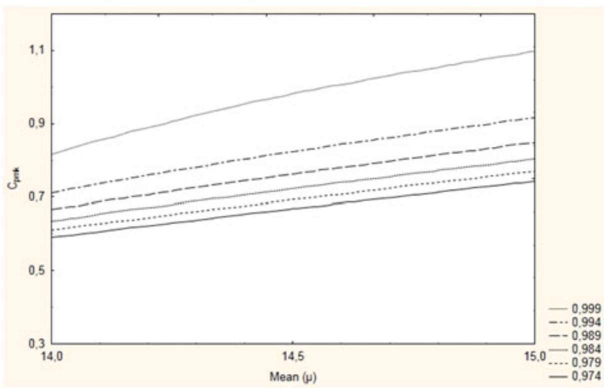

Figure 8. Contour plot of Equation (11) in the (μ, C_{pmk}) plane for large values of C_{pmk} and specifications given by $L = 10,\ U = 20$ and $T = M = 15$.

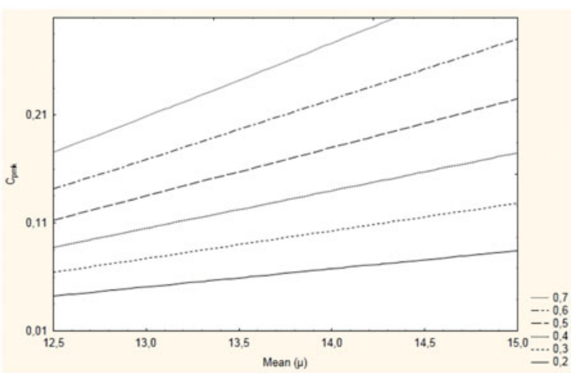

Figure 9. Contour plot of Equation (11) in the (μ, C_{pmk}) plane for small values of C_{pmk} and specifications given by $L = 10, U = 20$ and $T = M = 15$.

large or small values of C_{pmk} and process mean greater or smaller than the mid-point of the specification area. Therefore, the relationship of C_{pmk} to the proportion of conformance differs from that of C_{pm}, where, as shown previously, for small index values the proportion of conformance is maximized when the process mean is equal to the target value.

6. The proportion of conformance as a function of C_p, C_{pk}, C_{pm} and C_{pmk}

The proportion of conformance can be expressed as a function of the four basic PCIs, thus

$$p = \Phi\left[6C_p\left(1 - \frac{C_{pmk}}{C_{pm}}\right) + 3C_{pk}\right] - \Phi\left(-3\frac{C_pC_{pmk}}{C_{pm}}\right). \tag{16}$$

To show this, assume, without loss of generality, that $L \leq \mu \leq M$. (The proof is quite similar in the case where $M \leq \mu \leq U$). Then, the four indices take the

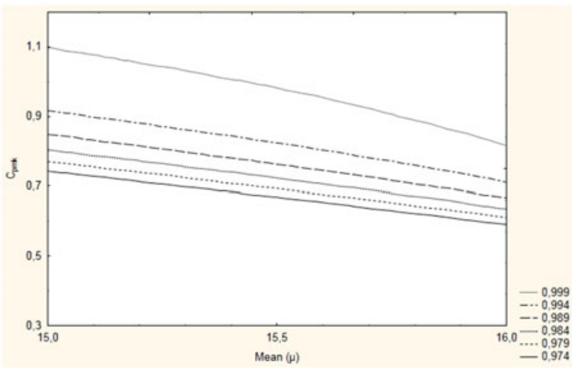

Figure 10. Contour plot of Equation (12) in the (μ, C_{pmk}) plane for large values of C_{pmk} and specifications given by $L = 10$, $U = 20$ and $T = M = 15$.

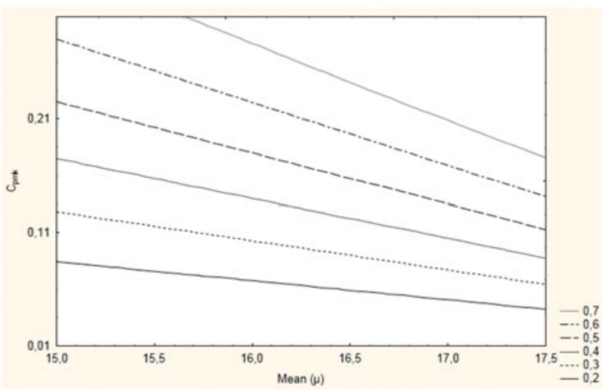

Figure 11. Contour plot of Equation (12) in the (μ, C_{pmk}) plane for small values of C_{pmk} and specifications given by $L = 10$, $U = 20$ and $T = M = 15$.

form

$$C_p = \frac{U-L}{6\sigma}, \quad C_{pk} = \frac{\mu - L}{3\sigma}, \quad C_{pm} = \frac{U-L}{6\sigma'} \quad \text{and} \quad C_{pmk} = \frac{\mu - L}{3\sigma'},$$

where $\sigma' = \sqrt{\sigma^2 + (\mu - \mathrm{T})^2}$. By using the definition of the proportion of conformance given in Equation (1), it suffices to show that

$$6C_p\left(1 - \frac{C_{pmk}}{C_{pm}}\right) + 3C_{pk} = \frac{U-\mu}{\sigma}. \tag{17}$$

and

$$-3\frac{C_pC_{pmk}}{C_{pm}} = \frac{L-\mu}{\sigma}. \tag{18}$$

Starting with Equation (17), one obtains

$$\begin{aligned} 6C_p\left(1 - \frac{C_{pmk}}{C_{pm}}\right) + 3C_{pk} &= \frac{U-L}{\sigma}\left(1 - \frac{2(\mu - L)}{U-L}\right) + \frac{\mu - L}{\sigma} \\ &= \frac{U-L}{\sigma} - \frac{2\mu - 2L}{\sigma} + \frac{\mu - L}{\sigma} = \frac{U-\mu}{\sigma}. \end{aligned}$$

Similarly, for Equation (18) we have

$$-3\frac{C_pC_{pmk}}{C_{pm}} = -3\frac{\frac{U-L}{6\sigma}\frac{\mu-L}{3\sigma'}}{\frac{U-L}{6\sigma'}} = -\frac{\mu - L}{\sigma} = \frac{L-\mu}{\sigma}.$$

Hence, Equation (16) has been established.

7. Concluding remarks

In this article, several formulae that connect the values of the four basic PCIs to that of the proportion of conformance were provided. The implications of the relationships established by these formulae are quite important in providing a deeper insight into the properties of the four basic PCIs. The most interesting findings are outlined below:

- The value of C_p is sufficient for obtaining an upper bound for p.
- The value of C_{pk} leads to an interval within which the value of p always lies.
- The value of the proportion of conformance is maximized, keeping the value of C_{pm} fixed, when $\mu = T$ provided that C_{pm} is very small.
- If the value of C_{pm} is sufficiently large, the value of p is minimized, for fixed C_{pm}, when $\mu = T$ and increases as the value of μ moves far from T to the direction of any of the specification limits.
- For any fixed value of the index C_{pmk}, the proportion of conformance is minimized when $\mu = T$ and increases as μ moves towards the specification limits.
- The proportion of conformance can be obtained from knowledge of the values of C_p and C_{pk}.

- The role of the value

$$2\Phi(3C) - 1 \tag{19}$$

  where C is one of the PCIs C_p, C_{pk}, C_{pm} and C_{pmk}, changes according to the selection of C. Actually, if $C = C_p$ then (19) gives the largest value that the proportion of conformance may take, while if $C = C_{pk}$ or $C = C_{pmk}$ then (19) determines a lower bound for p. Finally, if $C = C_{pm}$ then (19) determines a lower bound for p provided that the value of C_{pm} is sufficiently large. On the contrary, if the value of C_{pm} is very small then (19) provides an upper bound for p.
- The proportion of conformance can be expressed as a function of the four basic PCIs, according to Equation (16).

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