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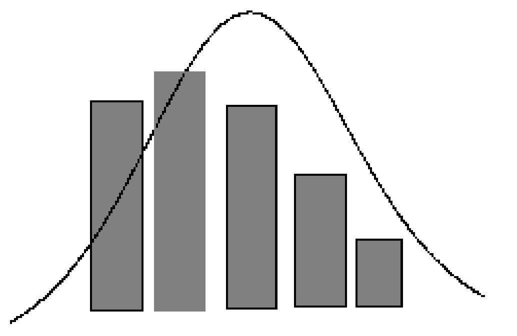

DEPARTMENT OF STATISTICS

TECHNICAL REPORT

How Heavy are Probability Tails of Wet and Dry Durations of Regionally Averaged Rain Fields ?

by

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ABSTRACT

Durations of rain events and drought events over a given region provide important information about the water resources of the region. Of particular interest is the shape of upper tails of the probability distributions of such durations, on the basis of which the return period of extreme events of drought or flooding may be assessed, for planning and management of water resources in a region of interest. Recent research has revealed certain multiscaling properties of sample tail quantiles of wet and dry durations of regionally averaged rain rate fields, suggesting that the underlying probability distributions of such durations have heavy tails of hyperbolic type, across a wide range of spatial scales from 2 *km* to 120 *km*. These findings are based on radar measurements of spatially averaged rain rate (SARR) over a tropical oceanic region. Using exactly the same data, the present work is concerned with formal statistical inference for the Pareto tail-index of wet and dry durations at each of those spatial scales. The approach taken is non-parametric, using state of the art procedures developed recently in the literature, including procedures based on the rather classical Hill estimator. The obtained results are discussed, comparing among the implemented procedures with respect to the estimates emerging from the multiscaling analysis.

KEYWORDS: Wet and dry durations of regional rainfall; Quantile multiscaling; Heavy tails; Pareto tail-index; Semi-parametric statistical inference.

1 Introduction

Rainfall is a physical phenomenon characterized by intermittency between dry and wet states across a very wide range of scales of observation, both spatial and temporal. This type of intermittency combined with the "wild" variability of rainfall intensity where and when it rains, from mild to moderate or higher and occasionally cataclysmic levels, have lead to the recognition of rain rate fields as multifractal structures [*Lovejoy and Mandelbrot* (1985), *Lovejoy and Schertzer* (1985, 1995), *Gupta and Waymire* (1990, 1993), *Tessier et al.* (1993), *Over and Gupta* (1994, 1996), *Marsan et al.* (1996), *Foufoula-Georgiou* (1998)]. Rainfall intensity is measured by *rain rate* in *mm/hr* units, representing the flux of water volume carried by rain droplets passing through (or landing on) an elementary surface of unitary area per unit of time. The dry state of rainfall is represented by zero rain rate and its wet state is represented by positive values of rain rate. If a field of rain rates is mapped over a (two-dimensional) geographic region at a given time instant, then the support of wet states constitutes a multifractal subset of the mapped region. Alternatively, if at each instant of time the observed rain rate field is spatially averaged over the region of interest, or if the region is as small as the orifice of a measuring rain-gauge, which for all practical purposes may be considered as a geometric point, then the support of wet states becomes a multifractal subset of the time interval during which rainfall is recorded. Dry epochs are defined as maximal time segments with zero rain rate (everywhere in the region), and wet epochs are defined as maximal time segments with positive rain rate (somewhere in the region). The lengths of such segments are referred to as dry and wet durations, respectively.

The shape of probability tails of durations of wet and dry epochs, in the regional sense just defined, is essential for several matters of practical importance as well as for issues of rather theoretical interest. For instance, assessing the return period or the frequency of extreme events, such as prolonged drought and prolonged raining epochs, is a matter of prime interest for provisional management of storage and consumption of water resources, especially in regions sensitive to or critical for water supplies. Having a sense of how heavy are the probability tails of wet and dry epoch durations is also important information for shaping realistic scenarios of stochastic rainfall forcing upon hydrological models used for regionalization of hydrologic extremes in large basins of river networks [*Gupta and Waymire* (1998), *Gupta* (2004)].

On rather theoretical grounds, empiricism pointing to hyperbolic probability tails of wet and dry epoch durations might be attributed to (or be justified by) the very perception of rain fields as intermittent multifractal structures [*Mandelbrot* (1983)]. As a matter of fact, the Pareto tail-index $(\Delta + 1)$ of dry durations is directly related with the (capacity) fractal dimension (Δ) of the (temporal) support of wet states [*Lowen and Teich* (1993), *Schmitt et al.* (1998), *Pavlopoulos and Gupta* (2003)]. Moreover, the probability of wet states (i.e. the probability of raining) in aggregated intermittent records or maps of rain rate, behaves as a power-law of the scale of aggregation, whose exponent is again determined by the fractal dimension of the initial (i.e. before aggregation) record or map [*Kedem and Chiu* (1987), *Over and Gupta* (1994), *Kundu and Bell* (2003)].

There is also emerging evidence that heavy enough tail probabilities (i.e. associated with small enough Pareto tail-index) of wet and dry epoch durations have an impact on the global memory properties of the underlying rainfall process, which might be observed or modelled at a given scale [*De Michele and Pavlopoulos* (2007)]. Indeed, because of their intermittency between wet and dry states, rainfall processes may be viewed from the perspective of alternating ON-OFF signals, whose memory properties are known to be influenced and in certain cases to be determined by the tail behavior of the distributions of ON (wet) and OFF (dry) epoch durations. In particular, heavy tailed distributions of OFF durations are often associated with the causes of long range dependence (LRD) [*Taqqu and Levy* (1986), *Willinger et al.* (1997), *Heath et al.* (1998), *Adler et al.* (1998), *Doukhan et al.* (2003), *Lowen and Teich* (2005)].

Motivated by the perception of rain fields as intermittent multifractal structures, *Pavlopoulos and Gupta* [2003] carried out a diagnostic study of scaling properties of tail quantiles of durations of wet and dry epochs of spatially averaged rain rate (SARR), with respect to the scale of the region over which rain rate is averaged. Among the main results of that study is that durations of regionally wet and dry epochs have probability tails of hyperbolic type, in all the probed spatial scales, with specific estimated values of the associated Pareto tail index corresponding to each spatial scale. The data used in that study and in the present work are briefly presented in the following Section 2, while the scaling properties of their sample quantiles are summarized in Section 3, merely for self-containment of the present work.

The purpose of this paper is to investigate whether the predicted estimates of Pareto tail index, via multiscaling analysis of wet and dry durations of SARR, provide acceptable or trustworthy indicators of tail heaviness, to the extent suggested by formal (i.e. model free) statistical procedures detached from any scaling properties. To this goal, we implement non-parametric tests of hypotheses, along with the corresponding confidence intervals and associated point estimators. Section 4 is dedicated to a brief presentation of a set of such procedures, which constitute state of the art methodology in the pertinent literature of semi-parametric and non-parametric statistical inference for Pareto tail index [*Cheng and Peng* (2001), *Jurečková* (2000, 2003), *Jurečková and Picek* (2001, 2004), *Fialová et al.* (2004), *Picek* (2006)]. In the final Section 5 we present the results obtained from the application of these procedures, organized in a sequence of four tables, and discuss those findings, comparing among the implemented procedures with respect to the estimates predicted by the multiscaling approach.

2 Working data and related issues

The raw data from which appropriate working data were formulated for the multiscaling analysis carried out by *Pavlopoulos and Gupta* [2003], is a time series of digital maps of radar reflectivity measurements. These maps were obtained during the Tropical Ocean Global Atmosphere (TOGA) Coupled Ocean-Atmosphere Response Experiment (COARE) by a shipboard Doppler precipitation radar (MIT). Each map corresponds to a single radar scan, probing a fixed oceanic region of reference S, with area 240x240 km^2, in the tropical sector of South Pacific Ocean (China Sea: 2^oS, 156^oE). The temporal resolution between successive scans is (approximately) 20 minutes. Reflectivity measurements Z from each scan, binned over square pixels of area $2{\times}2$ Km^2, have been converted to instantaneous rain rate R by the Z-R relationship $R = (Z/230)^{0.8}$, rendering a series of retrieved rain rate digital maps. The entire series corresponds to the full period of Cruise 1 (November 10, 1992 through December 9, 1992), consisting of 1992 scans, and to the early part of Cruise 2 (December 21, 1992 through December 29, 1992), consisting of 617 scans. A good source of detailed information about TOGA-COARE and its objectives is *Short et al.* [1997].

Because intermittency between wet and dry states was found to be rather limited in time series of SARR on the entire region of reference, as well as on square subregions of side length between 240 *km* and 120 *km*, multiscaling analysis of wet and dry durations has been restricted to spatial scales ranging from 120 *km* down to 2 *km*, following the rule of half (approximately). This amounts to a total of seven scales, 120, 60, 30, 16, 8, 4, 2 *km*, of which the largest is referred to as *scale of reference* and the smallest as *pixel scale*. Since spatial multiscaling addresses the dependence of a given quantitative characteristic (e.g. sample quantiles, moments, spectra, etc.) on the relative size or scale of geometrically similar subregions nested into one another, it is convenient to refer to spatial scales in terms of a unit-free *scale index* $\lambda \in (0, 1]$, formally defined as the ratio of diameters of two geometrically similar subregions, say A and A_λ, of which the smaller is nested inside the larger; i.e. $A_\lambda \subset A$. Thus, the scale index values corresponding to the herein considered spatial scales are $\lambda = 1$, 1/2, 1/4, 2/15, 1/15, 1/30, 1/60 respectively, relative to the scale of reference (120 *km*). For this set of scales, 25 different nestings of square (i.e. geometrically similar) subregions were sampled according to a certain symmetric design of spatial sampling. Due to its symmetry, that design covers densely the probed region S by subregions of each of the seven specified scales, minimizing overlaps between subregions of the same scale. Among the 25 nestings, the scale of reference is represented by only 5 different subregions, while each of the remaining six scales is represented by 25 different subregions. A time series of spatially averaged rain rate was obtained separately for Cruise 1 and Cruise 2 on each and every one of these 155 ($= 5 + 25 \times 6$) sampled sub regions. Spells of zeros and spells of positive values were identified as dry and wet epochs respectively. The integer-valued lengths of these spells, multiplied by 1/3, provide *"quantized"* working data of dry and wet durations in units of hours (*hr*). These estimates of duration presume the absence of any intermittency during the 20 minute sampling intervals between radar scans. This may be unrealistic, especially over regions of the smaller scales, and a source of positive bias. Moreover, because of a few blocks of missing scans incurring only 2.6% of missing values in every time series of SARR, wet and dry spells bordering with these blocks were discarded in order to avoid an extra source of ambiguity for the true length of wet and dry spells.

Quantization of working data of durations, combined with their extremely high skewness, makes scaling analysis of their sample quantiles impossible for probability levels $p < 0.8$ [see *Pavlopoulos and Gupta* (2001)]. On the other end, scaling analysis of sample *tail quantiles* for $p \geq 0.8$ relies heavily on extreme values in the top 20% range of working data. Since data from individual nestings provide very few extreme values in that range, especially in the larger scales, sample tail quantile estimates can be further biased, in addition to bias due to the quantized nature of the data. To suppress some of this bias, working data from both cruises were first pooled or merged on each sampled subregion, retaining chronological sequence. This step is referred to as *temporal pooling.* Then, temporally pooled data from all subregions of the same scale were also pooled, this step being referred to as *spatial pooling.* The final product of the overall pooling amounts to 14 sets (7 wet and 7 dry) of *spatio-temporally pooled* working data of durations. That is, a pair of wet and dry sets of duration data for each of the 7 scales considered.

It is important to remark that this spatio-temporal pooling strategy, implemented in order to enhance variability of extreme values in the top 20% range of the working data of wet and dry durations, is justifiable only under conditions that may guarantee invariance, stationarity, or homogeneity of probability distributions of durations (wet or dry distinctly) extracted from geometrically similar subregions of the same scale, throughout the duration of both cruises as well as across the entire region of reference S. For this cause, two assumptions of "Temporal Homogeneity" and "Spatial Homogeneity" have been postulated and subsequently verified as being statistically significant via non-parametric testing procedures in *Pavlopoulos and Gupta* [2003, Section 4].

3 Multiscaling of tail quantiles

In light of the underlying homogeneity assumptions, spatio-temporally pooled working data of wet durations and (separately of) dry durations, from square subregions of a given spatial scale with index $0 < \lambda \leq 1$ relative to the scale of reference, may well be considered as samples drawn from the laws of random variables W_λ and D_λ, respectively, with $Q_\lambda^{(w)}(p)$ and $Q_\lambda^{(d)}(p)$ denoting the corresponding quantile functions. Relying on sample tail-quantiles obtained on a grid of 10 probability levels $0.8 \leq p \leq 0.995$, for each of the 14 sets of spatio-temporally pooled working data, and combining results from a series of regressions of logarithms of sample tail-quantiles against scale index λ and probability level p, *Pavlopoulos and Gupta* [2003] formulated the following parametric models for *tail quantile functions* of wet and dry epoch durations, respectively,

$$Q_\lambda^{(w)}(p) = e^{\alpha \ln \lambda + \beta} \cdot (1-p)^{\gamma \ln \lambda + \delta}, \tag{1}$$

$$Q_\lambda^{(d)}(p) = e^{\alpha^* \lambda + \beta^*} \cdot (1-p)^{\gamma^* \lambda + \delta^*}, \tag{2}$$

where $\alpha = 0.3652$, $\beta = 0.8746$, $\gamma = 0.0285$, $\delta = -0.5006$, $\alpha^* = -0.5117$, $\beta^* = 0.2327$, $\gamma^* = 0.379$, $\delta^* = -0.57$ are estimates of parameter values obtained through regression.

Dividing (1) and (2) by their corresponding instances at the scale of reference (i.e. at $\lambda = 1$), it is seen that the scaling of wet tail-quantiles is of power-law type,

$$Q_\lambda^{(w)}(p) = \lambda^{\alpha + \gamma ln(1-p)} Q_1^{(w)}(p), \tag{3}$$

while the scaling of dry tail-quantiles is of exponential type,

$$Q_\lambda^{(d)}(p) = e^{[\alpha^* + \gamma^* ln(1-p)](\lambda - 1)} Q_1^{(d)}(p), \tag{4}$$

with respect to λ for a given tail probability level p (i.e. for p near 1). The asymmetry between *power-law multiscaling* of wet tail-quantiles and *exponential multiscaling* of dry tail-quantiles is discussed in some detail by *Pavlopoulos and Gupta* [2003]. Figures 1-2 depict QQ-plots between sample tail quantiles and predicted tail quantiles according to the multiscaling models (1) and (2), respectively. Both models constitute quite significant improvements when compared against power-law *simple scaling* models (i.e. $\gamma = 0$ in (3)). Simple scaling for both wet and dry duration quantiles has been investigated as a potentially valid theory in rather limited ranges of small scales on the basis of TOGA-COARE releases [*Gritsis* (1997), *Pavlopoulos and Gritsis* (1999), *Pavlopoulos and Gupta* (2001)].

Inverting formulae (1) and (2) one easily obtains (upper) tail probabilities of wet durations

$$P\left(W_\lambda > u\right) = e^{-(\alpha \ln \lambda + \beta)/(\gamma \ln \lambda + \delta)} \cdot u^{1/(\gamma \ln \lambda + \delta)}, \tag{5}$$

for $u \geq Q_\lambda^{(w)}(0.8) = e^{\alpha \ln \lambda + \beta} \cdot 0.2^{\gamma \ln \lambda + \delta}$, and (upper) tail probabilities of dry durations

$$P\left(D_\lambda > u\right) = e^{-(\alpha^* \lambda + \beta^*)/(\gamma^* \lambda + \delta^*)} \cdot u^{1/(\gamma^* \lambda + \delta^*)}, \tag{6}$$

for $u \geq Q_\lambda^{(d)}(0.8) = e^{\alpha^* \lambda + \beta^*} \cdot 0.2^{\gamma^* \lambda + \delta^*}$. Formulae (5) and (6) not only reveal that durations of wet and dry epochs have hyperbolic tails in all the spatial scales considered in the scaling analysis, but also provide specific estimates of the Pareto tail index corresponding to each scale. These estimates are given by $m_0(\lambda) = -(\gamma \ln \lambda + \delta)^{-1}$ for wet tails, and by $m_0^*(\lambda) = -(\gamma^* \lambda + \delta^*)^{-1}$ for dry tails, as reported in each of Tables 1-4 immediately under the rows reporting sample sizes (ℓ) of spatio-temporally pooled working data from square subregions of the same scale.

According to these estimates, the tail index of both wet and dry durations increases with respect to the scale index λ, indicating that tails might be heavier in smaller regions than in larger ones. However, across scales greater than 8 *km* the rate at which dry tail index increases seems appreciably steeper than that of the wet tail index, while in the rather short range of scales between 2 and 8 *km* the relative increments of wet tail index exceed those of the dry tail index. Yet, tails of wet duration appear to be potentially heavier than tails of dry duration at each given scale of observation. It is also noteworthy that estimates $m_0(\lambda)$ indicate the possibility of the duration of wet epochs having heavy enough tails, so that second moments may not exist at any of the probed scales, while the estimates $m_0^*(\lambda)$ restrict the possibility of infinite variance for the duration of dry epochs only in regions of smaller scale (e.g. below 30 *km*).

4 Semi-parametric procedures of inference for Pareto tail index

4.1 Definition of the Pareto tail index

A random variable or its probability distribution is said to be *heavy tailed* if and only if its cumulative distribution function (CDF), denoted by F, fulfills the asymptotic condition

$$\lim_{u \to \infty} \frac{-\log(1 - F(u))}{m \log u} = 1, \tag{7}$$

for some $0 < m < \infty$. A typical example of a distribution satisfying (1) is the Pareto distribution, with tail probabilities $1 - F(u) = u^{-m} I[u \geq 1]$. For this reason, the number m in (7) is referred to as the *Pareto tail index* of F. This terminology is also justifiable by the fact that (7) is equivalent to having tail probabilities of the form [see *Embrechts et al.* (1997), Theorem 3.3.7]

$$1 - F(u) = u^{-m} L(u), u \in \mathbb{R}, \tag{8}$$

with L being a positive function slowly varying at infinity; i.e. $\lim_{u\to\infty} \frac{L(au)}{L(u)} = 1$, for all $a > 0$. Therefore, heavy tailed distributions are characterized as members of the Fréchet maximum domain of attraction (MDA), with parameter $\xi = 1/m > 0$ in the von Mises representation of generalized extreme value distributions (GEV).

In this paper our interest is focused partly on obtaining confidence intervals and partly on testing hypotheses for the Pareto tail index. Both of these subjects have received rather limited attention in the existing literature for extreme-value statistics, despite the great momentum of research developed in this area of statistics over the recent three decades or so.

4.2 Inference based on the Hill estimator

Several point estimators have been proposed for the tail index of a heavy-tailed distribution [e.g. see *Hill* (1975), *Pickands* (1975), *Dekkers et al.* (1989)]. However, the Hill estimator remains by far the most popular one, perhaps due to its computational simplicity and a handful of convenient interpretations it admits, based on the idea of using the upper $k+1$ order statistics from a random sample of size $\ell > k$. Given a random sample of i.i.d. random variables $X_1, \dots, X_\ell$, for each $1 < k < \ell$ the *Hill estimator* is defined by

$$H(k) = \frac{1}{k} \sum_{i=1}^{k} \log X_{(\ell-i+1:\ell)} - \log X_{(\ell-k:\ell)},$$

where $X_{(i:\ell)}$ denotes the i-th order statistic of the given sample. For small k the variance of $H(k)$ becomes large, while large values of k introduce substantial bias in the estimation. Therefore, the choice of the *sample fraction* k plays an important role in the properties of the estimator. Consequently, any inference based on such estimators is genuinely *semi-parametric.* Some general rules of thumb for choosing k have been proposed by *Boos* [1984], $k = 0.2\ell$ for $50 \le \ell \le 500$ or $k = 0.1\ell$ for $500 < \ell \le 5000$, and by *Castillo et al.* [1989], $k = 2\sqrt{\ell}$.

The standard approach to obtain confidence intervals for the tail index is by exploiting asymptotic normal approximations of the Hill estimator's distribution. In this spirit, *Cheng and Pan* [1998] considered a one-term Edgeworth expansion of the distribution function of the Hill estimator in cases where the asymptotic bias is zero, while *Cheng and Peng* [2001] provided an algorithm for computing a *plug-in* value of the theoretically optimal sample fraction k^*, in the sense of minimizing *absolute coverage error* of the confidence interval. A key assumption, instrumental for the derivation of this optimal sample fraction k^*, has been that the scalar function L is of a specific form, such that the underlying heavy tailed distribution corresponds to a special case of second-order regular variation [see *de Haan and Stadtmüller* (1996)].

The two-sided confidence interval for the Pareto tail index $m = 1/\xi$ obtained by this approach, at level $0 < \alpha < 1$, is given by

$$I_2(\alpha, k) = \left(\frac{\sqrt{k}}{\sqrt{k}H(k) + \Phi^{-1}(1-\alpha/2)H(k)} \;, \frac{\sqrt{k}}{\sqrt{k}H(k) - \Phi^{-1}(1-\alpha/2)H(k)} \right),$$

while one-sided right and left intervals are respectively given by

$$I_1(\alpha, k) = \left(\frac{\sqrt{k}}{\sqrt{k}H(k) + \Phi^{-1}(1-\alpha)H(k)}, \infty \right) \& \left(0, \frac{\sqrt{k}}{\sqrt{k}H(k) - \Phi^{-1}(1-\alpha)H(k)} \right),$$

where Φ denotes the CDF of the standard normal law. These are also referred to as *Wald* intervals, among several other types of intervals reviewed in a recent paper by *Haeusler and Segers* [2007] elaborating some new developments on Hill-based confidence intervals for $\xi = 1/m$ from Edgeworth expansions under certain asymptotic conditions on the bias.

4.3 Inference based on sub-sample statistics

Considering the estimates $m_0(\lambda)$ and $m_0^*(\lambda)$ as benchmark values of the Pareto tail index associated with hyperbolic tails of wet and dry durations of SARR, according to the multiscaling parametric models (1) and (2) for tail quantiles, it is natural to wonder how accurate or reliable these estimates may be. There are several reasons for such skepticism, considering the small number of scales (only seven) and the small number of tail probability levels (only ten) upon which all simple linear regressions were executed, in the ordinary least squares sense. A sensible way to enlighten such skepticism, is to formulate hypotheses of the form $\mathbf{H}_{m_0} : m \leq m_0$ (or $\mathbf{H}^*_{m_0} : m \geq m_0$) and to test them against one-sided alternatives of the form $\mathbf{K}_{m_0} : m > m_0$, (or $\mathbf{K}^*_{m_0} : m < m_0$, respectively), for fixed $m_0 > 0$ provided by the predicted benchmark values $m_0(\lambda)$ and $m_0^*(\lambda)$ of the wet and dry tail index via multiscaling.

Nonparametric procedures for testing exactly this type of hypotheses about the Pareto tail index of an unknown heavy tailed distribution have been developed rather recently by *Jurečková* (2000, 2003), *Jurečková and Picek* (2001), *Picek and Jurečková* (2001). The key idea in this approach is to assume that a sample drawn from a heavy tailed distribution, may somehow be considered as a set of N independent sub-samples $\mathbf{X}_j = (X_{j1}, \dots, X_{jn})'$, for $j = 1, \dots, N$, each comprised of a fixed number of i.i.d. observations, n. Then, consistent and asymptotically normal procedures can be obtained for testing one-sided hypotheses of the above types, based on the empirical CDF of certain statistics computed across the available population of sub-samples. Specific statistics for which this theory is fully developed in the cited references are:

Sub-Sample Maxima: $X_{(n)}^{(j)} = \max\{X_{j1}, \ldots, X_{jn}\}$, $j = 1, \ldots, N$, with empirical CDF

$$\hat{F}_N^*(u) = \frac{1}{N} \sum_{j=1}^{N} I[X_{(n)}^{(j)} \leq u]$$

,

Sub-Sample Averages: $\bar{X}_n^{(j)} = \frac{1}{n} \sum_{i=1}^{n} X_{ji}$, $j = 1, \ldots, N$, with empirical CDF

$$\hat{F}_N^{**}(u) = \frac{1}{N} \sum_{j=1}^{N} I[\bar{X}_n^{(j)} \leq u]$$

.

Sub-Sample Averaged Block Maxima: $\hat{\theta}_n^{(j)} = (X_j^{(1)} + X_j^{(2)})/4$, with empirical CDF

$$\hat{F}_N^{***}(u) = \frac{1}{N} \sum_{j=1}^{N} I[\hat{\theta}_n^{(j)} \leq u],$$

where $X_j^{(1)} = X_{(\nu)}^{(j)} = \max\{X_{j1}, \ldots, X_{j\nu}\}$, $X_j^{(2)} = \max\{X_{j(\nu+1)}, \ldots, X_{jn}\}$, for $j = 1, \ldots, N$ and some fixed $1 \leq \nu \leq n - 1$.

In particular, the following decision rules apply when statistics of sub-sample maxima are used; see *Jurečková* (2000, 2003), *Jurečková and Picek* (2001), *Picek and Jurečková* (2001):

Reject $\mathbf{H}_{m_0} : m \leq m_0$ against $\mathbf{K}_{m_0} : m > m_0$, at the asymptotic significance level $0 < \alpha < 1$, when

$$\textbf{either } 1 - \hat{F}_N^*(u_{N,m_0}) = 0,$$

$$\textbf{or } 1 - \hat{F}_N^*(u_{N,m_0}) > 0 \ \text{ and}$$

$$N^{\delta/2}\left[-\log(1 - \hat{F}_N^*(u_{N,m_0})) - (1-\delta)\log N\right] \geq \Phi^{-1}(1-\alpha),$$

where $u_{N,m} := (nN^{1-\delta})^{\frac{1}{m}}$, for a chosen constant $0 < \delta < \frac{1}{2}$, and

Reject $\mathbf{H}_{m_0}^* : m \geq m_0$ against $\mathbf{K}_{m_0}^* : m < m_0$, when

$$\textbf{either } \hat{F}_N^*(u_{N,m_0}) = 0,$$

$$\textbf{or } \hat{F}_N^*(u_{N,m_0}) > 0 \ \text{ and}$$

$$N^{\delta/2}\left[-\log(1 - \hat{F}_N^*(u_{N,m_0})) - (1-\delta)\log N\right] \leq \Phi^{-1}(\alpha).$$

From these decision rules one-sided confidence intervals can be obtained in terms of empirical quantiles $\hat{F}_N^{*-1}(u) = \inf\{s : \ \hat{F}_N^*(s) > u\}$; see *Picek* (2006). The right-sided interval is

$$J_1(\alpha,\delta) = \left(\frac{\log(nN^{1-\delta})}{\log\left(\hat{F}_N^{*-1}\left(1 - \exp\left\{-N^{-\delta/2}\Phi^{-1}(1-\alpha) - (1-\delta)\log N\right\}\right)\right)} \ , \ \infty \right)$$

and the left-sided interval is

$$J_1(\alpha,\delta) = \left(0 \quad , \quad \frac{\log(nN^{1-\delta})}{\log\left(\hat{F}_N^{*-1}\left(1-\exp\left\{N^{-\delta/2}\Phi^{-1}(1-\alpha)-(1-\delta)\log N\right\}\right)\right)}\right).$$

Moreover, by inverting the test procedures in the Hodges-Lehman sense, one obtains strongly consistent point estimators of m given by

$$M^*(\delta) = \left(M_+^* + M_-^*\right)/2,$$

where $M_+^* := \sup\{m :\ 1-\hat{F}_N^*(u_{N,m}) < N^{-(1-\delta)}\}$ and $M_-^* := \inf\{m :\ 1-\hat{F}_N^*(u_{N,m}) > N^{-(1-\delta)}\}$; see *Jurečková and Picek* (2004).

All the above formulae for one-sided testing, one-sided confidence intervals and point estimation of tail index, remain valid when the empirical CDF $\hat{F}_N^*$ of sub-sample maxima is replaced by $\hat{F}_N^{**}$ or $\hat{F}_N^{***}$, as long as $u_{N,m}$ is also modified accordingly, from $(nN^{1-\delta})^{\frac{1}{m}}$ to $N^{(1-\delta)/m_0}$ when sub-sample averages are used, or to $N^{(1-\delta)/m}$ when sub-sample averaged block-maxima are used.

It is worthy to remark that this framework cannot address two-sided alternative hypotheses or two-sided confidence intervals for the tail index, because the slowly varying scalar function L influences the variance of the asymptotic normal law of the test statistic $M^*(\delta)$ in so far uncontrolled ways. Although this might be considered as a shortcoming, it is also an advantage, because the obtained testing criteria and the corresponding one-sided intervals are applicable quite generally, not limited by restrictions such as second-order regular variation, whatsoever. Indeed, extensive simulation studies have shown that these tests distinguish very well the tail behavior among different types of distributions, even for moderate sample sizes. In particular, these procedures perform best and may be considered as more reliable, when based on sub-sample maxima instead of the other two sub-sample statistics. Yet, the performance of these procedures is generally influenced by the chosen value of the constant δ, rendering the entire setting genuinely semi-parametric again. This constant should be distinguished from and not be confused with the regression parameter δ in (1).

5 Results and discussion

Here we present the numerical results obtained from the procedures described in Section 4, as applied to each of the 14 sets of spatio-temporally pooled working data of wet and dry durations of SARR across the considered seven spatial scales of averaging. As already mentioned, our focus of interest is mainly on confidence intervals and on testing of one-sided hypotheses, about the Pareto tail index m of the underlying probability distributions of wet and dry durations at each spatial scale. Tail heaviness of these distributions is an assumption adopted in light of the evidence established by the multiscaling analysis of sample tail quantiles of these very data. The i.i.d. assumption among spatio-temporally pooled working data (wet or dry separately) at each scale, required by the intended inference procedures, is also adopted in light of sufficient evidence provided by the non-parametric testing of temporal and spatial homogeneity assumptions given in *Pavlopoulos and Gupta* [2003, Section 4]. In particular, independence among sub-samples of fixed small size $n = 4$ has been furnished at each scale by an appropriate scheme of random sampling. Results based on all three kinds of sub-sample statistics have been obtained, but only those from the use of sub-sample maxima are shown here, primarily due to space limitations, but also because they do not alter the general conclusions that we derive from comparisons based on the following Tables 1-4.

Table 1 tabulates P-values for testing one-sided hypotheses $\mathbf{H}_{m_0}$ and $\mathbf{H}^*_{m_0}$, when m_0 equals the predicted (via multiscaling analysis) benchmark values $m_0(\lambda)$ and $m^*_0(\lambda)$ corresponding to wet and dry tail index. P-values marked by $*$ indicate cases where the Pareto tail index is inferred to not exceed these benchmark values. In these cases, tails are inferred to be at least as heavy as predicted by the multiscaling analysis of sample tail quantiles, in the sense that either $\mathbf{H}_{m_0}$ is *not* rejectable (i.e. P-value greater than 0.01) or $\mathbf{H}^*_{m_0}$ *is* rejectable (i.e. P-value less than 0.01). Pairs of P-values indicating that *both* $\mathbf{H}_{m_0}$ and $\mathbf{H}^*_{m_0}$ are *not* rejectable (i.e. both P-values exceeding 0.01), for durations (wet or dry) from the same scale and for the same value of the semi-parameter δ, are marked in boldface to indicate admissibility of the corresponding predicted benchmark value m_0. For example, wet durations at the 4 *km* scale for $\delta = 0.25$ give P-value 0.982 to $\mathbf{H}_{1.67} : m \leq 1.67$ and P-value 0.018 to $\mathbf{H}^*_{1.67} : m \geq 1.67$, indicating that the formal intersection $\mathbf{H}_{1.67} \bigcap \mathbf{H}^*_{1.67} : m = 1.67$ may (at least heuristically) be considered non-rejectable at the 0.01 level of significance. In this sense, the predicted benchmark value 1.67 is considered *admissible.*

Tables 2 and 3 tabulate right and left (respectively) one-sided confidence intervals for the Pareto tail index of wet and dry epoch durations, at 95% confidence level (i.e. for $\alpha = 0.05$). The $J_1(0.05)$ intervals have been obtained by inverting the criteria for testing $\mathbf{H}_{m_0} : m \leq m_0$ in Table 2, and for testing $\mathbf{H}^*_{m_0} : m \geq m_0$ in Table 3, based on empirical CDF's of sub-sample maxima (i.e. based on F^*), while the $I_1(0.05)$ intervals are based on the Hill estimator for three different options of choosing the sample fraction k. Intervals containing the corresponding benchmark values of Pareto tail index m_0 predicted via multiscaling analysis of sample tail quantiles are marked with $*$, rendering those values non-refutable. However, unmarked intervals indicate that the values m_0 predicted by multiscaling might underestimate (Table 2) or overestimate (Table 3) the true index of heavy tailed durations, with probability not greater than 5% for each of these two events.

Table 4 tabulates values of the point estimator $M^*(\delta)$ and the Hill estimator $H(k)$, for the Pareto tail index of wet and dry epoch durations, along with two-sided 95% confidence intervals $I_2(0.05)$ based on the Hill estimator. Intervals containing the estimated values of Pareto tail index m_0 by multiscaling are again $*$-marked. Within each bin of point estimates, the value being "closest" to the corresponding value m_0 predicted by multiscaling is marked in boldface.

Overall, we conclude that to a large extent there is good agreement between the herein implemented state of the art semi-parametric procedures of statistical inference for the Pareto tail index, and its predicted values via the multiscaling regression approach, for both wet and dry durations across all the considered spatial scales. This is quite reassuring, as much for the competence of the multiscaling approach, as for the performance of the implemented semi-parametric procedures, which to the extent of our awareness have not been applied to any real data before, but exclusively to simulated data.

As a closing remark it is worthy to emphasize that the data used for this study represent the climate of a tropical oceanic region during the monsoon period, which is very rich in rainfall represented by long wet epochs, during which extremely high rain rates are locally experienced due to predominantly convective clouds. However, tail heaviness of *both* wet and dry epochs might be challenged in other types of climate. For instance, in sub-tropical, temperate, or even semiarid regions of complex geography (instead of just ocean all around), possibly interacting with ecosystems residing therein, the distributions of endurance of wet and dry epochs may be affected so that, if at all heavy tailed, the corresponding tail index might reflect such climatic or environmental dependencies. This prompts for further testing of theories for heavy tailed durations in regional rainfall, subject to availability of long (longer than monthly span) and with high frequency time series of reliable mappings of rain fields retrieved from radar or satellite probes.

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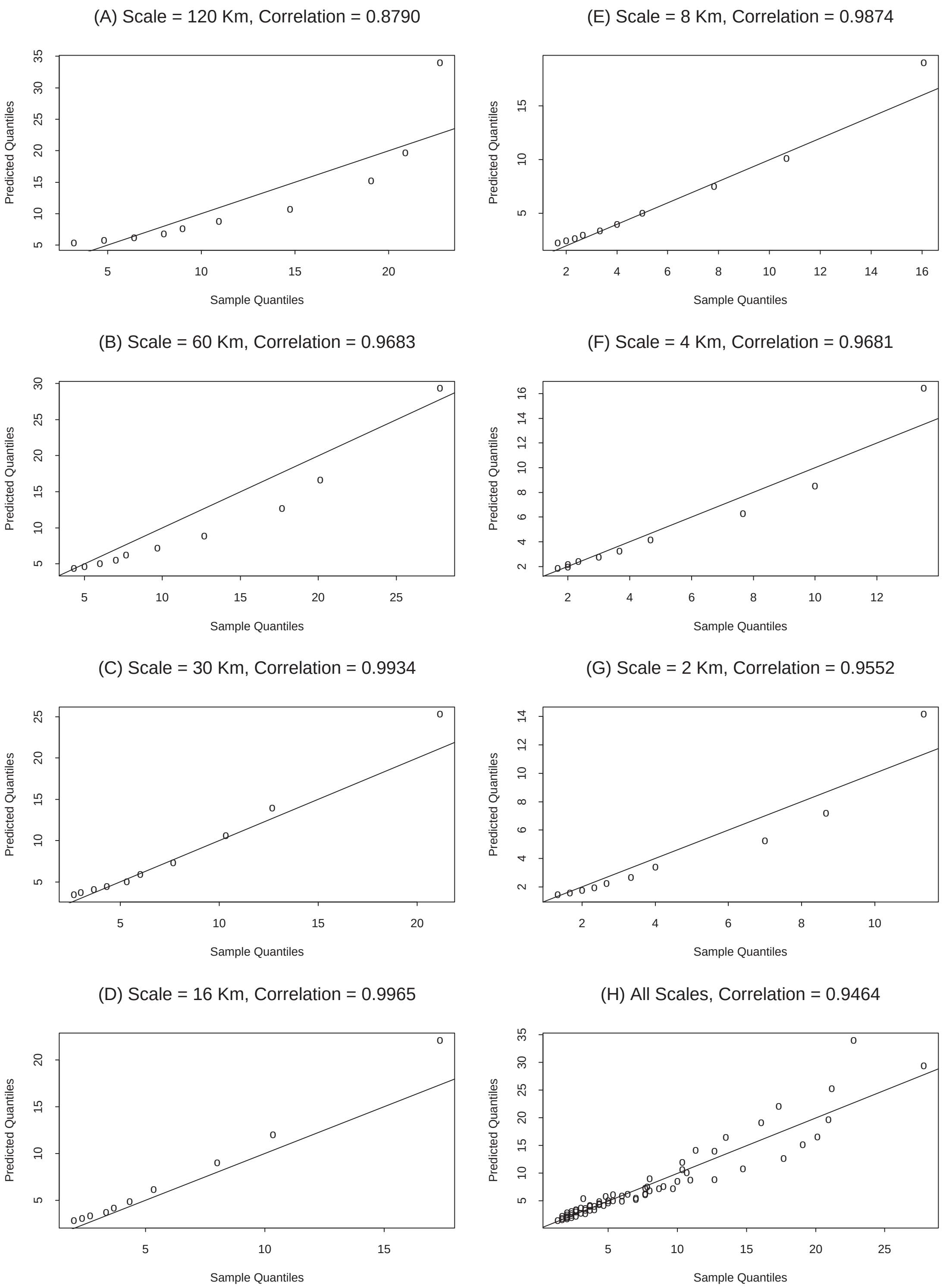

Figure 1: Q-Q-plots of wet duration quantiles predicted by the *power-law* multiscaling model (1), versus sample estimates of wet duration quantiles, at probability levels 0.8, 0.825, 0.85, 0.875, 0.9, 0.925, 0.95, 0.975, 0.985, 0.995, for each of the seven scales considered (A-G plots) and collectively across all scales combined (H plot). The correlation coefficient reported on each plot was obtained from simple linear regression of predicted tail-quantiles against their sample estimates. The line drawn in each plot is the diagonal through the origin.

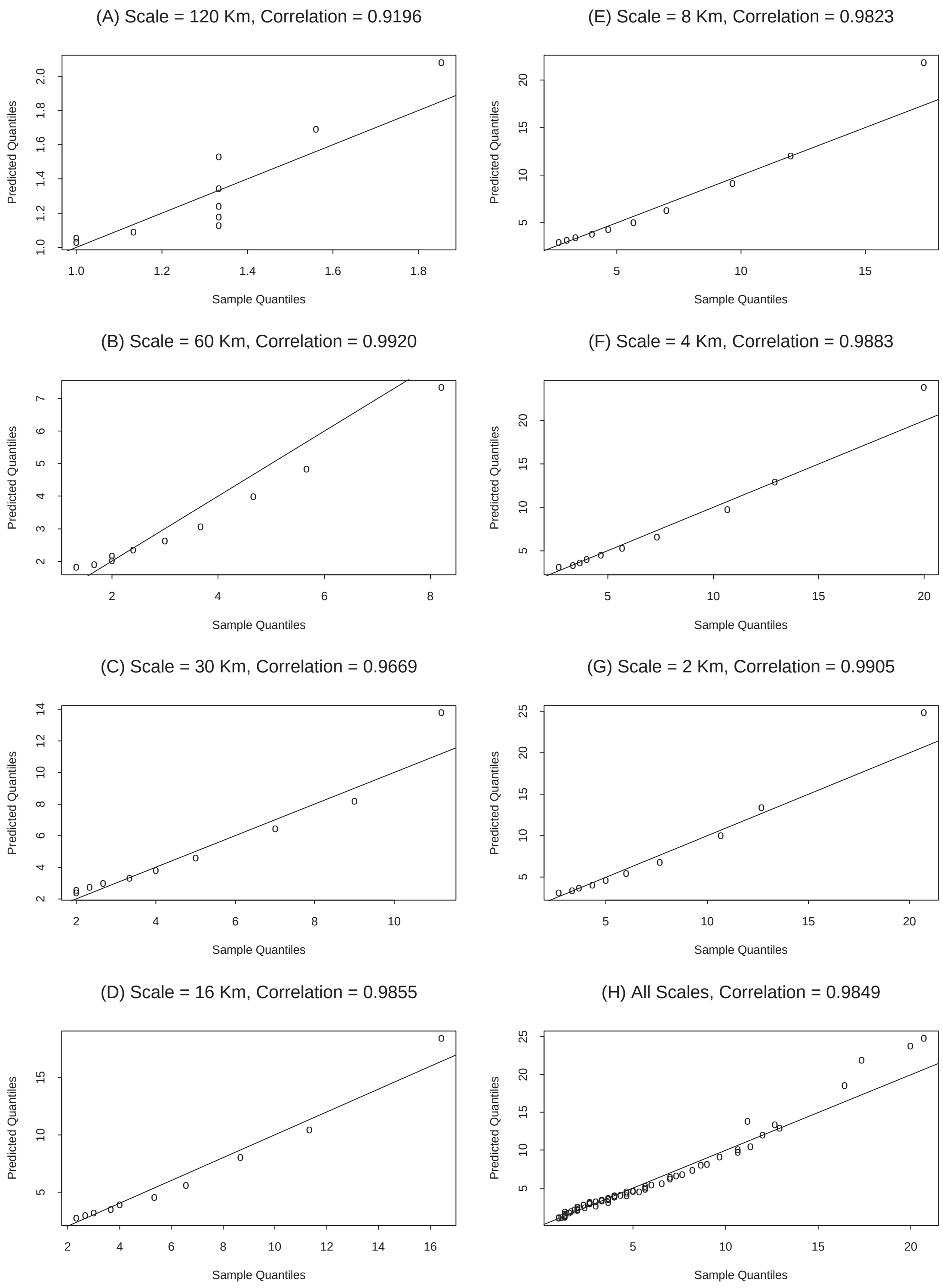

Figure 2: Q-Q-plots of dry duration quantiles predicted by the *exponential* multiscaling model (2), versus sample estimates of dry duration quantiles, at probability levels 0.8, 0.825, 0.85, 0.875, 0.9, 0.925, 0.95, 0.975, 0.985, 0.995, for each of the seven scales considered (A-G plots) and collectively across all scales combined (H plot). The correlation coefficient reported on each plot was obtained from simple linear regression of predicted tail-quantiles against their sample estimates. The line drawn in each plot is the diagonal through the origin.

Scale of spatial averaging	2 *km*	4 *km*	8 *km*	16 *km*	30 *km*	60 *km*	120 *km*
Wet durations sample size (ℓ)	2824	2897	2983	3179	3038	1421	25
Tail-index $m_0 = m_0(\lambda)$	1.62	1.67	1.73	1.79	1.85	1.92	1.99
$\mathbf{H}_{m_0}$ P-values, $\delta = 0.05$	0.000	0.000	* **0.666**	* **0.893**	* 0.998	* 1.000	* **0.852**
$\mathbf{H}_{m_0}$ P-values, $\delta = 0.25$	* **0.110**	* **0.982**	* 1.000	* 1.000	* 1.000	* 1.000	* **0.865**
$\mathbf{H}_{m_0}$ P-values, $\delta = 0.45$	* 1.000	* 1.000	* 1.000	* 1.000	* 1.000	* 1.000	* **0.743**
$\mathbf{H}^*_{m_0}$ P-values, $\delta = 0.05$	1.000	1.000	**0.334**	**0.107**	* 0.002	* 0.000	**0.148**
$\mathbf{H}^*_{m_0}$ P-values, $\delta = 0.25$	**0.890**	**0.018**	0.000	* 0.000	* 0.000	* 0.000	**0.135**
$\mathbf{H}^*_{m_0}$ P-values, $\delta = 0.45$	* 0.000	* 0.000	* 0.000	* 0.000	* 0.000	* 0.000	**0.257**
Dry durations sample size (ℓ)	2530	2618	2789	3087	3145	1679	45
Tail-index $m_0 = m^*_0(\lambda)$	1.77	1.79	1.83	1.92	2.10	2.62	5.23
$\mathbf{H}_{m_0}$ P-values, $\delta = 0.05$	*0.993	*0.990	***0.958**	*0.990	***0.986**	*1.000	***0.986**
$\mathbf{H}_{m_0}$ P-values, $\delta = 0.25$	*1.000	*1.000	*1.000	*1.000	*1.000	*1.000	*0.992
$\mathbf{H}_{m_0}$ P-values, $\delta = 0.45$	*1.000	*1.000	*1.000	*1.000	*1.000	*1.000	***0.987**
$\mathbf{H}^*_{m_0}$ P-values, $\delta = 0.05$	*0.007	* 0.010	**0.042**	*0.010	**0.014**	*0.000	**0.014**
$\mathbf{H}^*_{m_0}$ P-values, $\delta = 0.25$	*0.000	* 0.000	*0.000	*0.000	*0.000	*0.000	*0.008
$\mathbf{H}^*_{m_0}$ P-values, $\delta = 0.45$	*0.000	* 0.000	*0.000	*0.000	*0.000	*0.000	**0.013**

Table 1: P-values for testing one-sided hypotheses $\mathbf{H}_{m_0} : m \leq m_0$ (vs. $\mathbf{K}_{m_0} : m > m_0$) and $\mathbf{H}^*_{m_0} : m \geq m_0$ (vs. $\mathbf{K}^*_{m_0} : m < m_0$) for the Pareto tail index of wet and dry epoch durations.

Scale of spatial averaging	2 km	4 km	8 km	16 km	30 km	60 km	120 km
Wet durations sample size (ℓ)	2824	2897	2983	3179	3038	1421	25
Tail-index $m_0 = m_0(\lambda)$	1.62	1.67	1.73	1.79	1.85	1.92	1.99
$J_1(0.05)$, $\delta = 0.05$	$(1.750, \infty)$	$(1.753, \infty)$	$*(1.705, \infty)$	$*(1.590, \infty)$	$*(1.691, \infty)$	$*(1.451, \infty)$	$*(0.760, \infty)$
$J_1(0.05)$, $\delta = 0.25$	$*(1.569, \infty)$	$*(1.468, \infty)$	$*(1.517, \infty)$	$*(1.436, \infty)$	$*(1.425, \infty)$	$*(1.255, \infty)$	$*(0.669, \infty)$
$J_1(0.05)$, $\delta = 0.45$	$*(1.406, \infty)$	$*(1.337, \infty)$	$*(1.281, \infty)$	$*(1.280, \infty)$	$*(1.211, \infty)$	$*(1.063, \infty)$	$*(0.587, \infty)$
$I_1(0.05)$, $k =$ plug-in k^*	$*(1.231,\infty)$	$*(1.449, \infty)$	$(1.859, \infty)$	$*(1.599, \infty)$	$(2.563, \infty)$	$(2.441, \infty)$	$*(0.713, \infty)$
$I_1(0.05)$, $k = \ell/10$	$*(1.538,\infty)$	$*(1.545, \infty)$	$*(1.629, \infty)$	$*(1.652, \infty)$	$(1.964, \infty)$	$*(1.579, \infty)$	$*(0.408, \infty)$
$I_1(0.05)$, $k = 2\sqrt{\ell}$	$(1.731,\infty)$	$*(1.666, \infty)$	$(1.803, \infty)$	$*(1.776, \infty)$	$(2.305, \infty)$	$(2.224, \infty)$	$*(0.390, \infty)$
Dry durations sample size (ℓ)	2530	2618	2789	3087	3145	1679	45
Tail-index $m_0 = m_0^*(\lambda)$	1.77	1.79	1.83	1.92	2.10	2.62	5.23
$J_1(0.05)$, $\delta = 0.05$	$*(1.660, \infty)$	$*(1.668, \infty)$	$*(1.693, \infty)$	$*(1.770, \infty)$	$*(1.820, \infty)$	$*(1.909, \infty)$	$*(2.185, \infty)$
$J_1(0.05)$, $\delta = 0.25$	$*(1.425, \infty)$	$*(1.432, \infty)$	$*(1.484, \infty)$	$*(1.494, \infty)$	$*(1.594, \infty)$	$*(1.740, \infty)$	$*(2.025, \infty)$
$J_1(0.05)$, $\delta = 0.45$	$*(1.191, \infty)$	$*(1.202, \infty)$	$*(1.263, \infty)$	$*(1.283, \infty)$	$*(1.421, \infty)$	$*(1.482, \infty)$	$*(1.986, \infty)$
$I_1(0.05)$, $k =$ plug-in k^*	$(2.365, \infty)$	$(2.046, \infty)$	$(2.342, \infty)$	$(2.778, \infty)$	$(2.844, \infty)$	$*(2.163, \infty)$	$*(2.719, \infty)$
$I_1(0.05)$, $k = \ell/10$	$*(1.720, \infty)$	$*(1.601, \infty)$	$*(1.784, \infty)$	$*(1.573, \infty)$	$*(1.791, \infty)$	$*(2.294, \infty)$	$*(2.403, \infty)$
$I_1(0.05)$, $k = 2\sqrt{\ell}$	$(1.981, \infty)$	$(1.860, \infty)$	$(2.215, \infty)$	$(2.103, \infty)$	$(2.360, \infty)$	$*(2.285, \infty)$	$*(1.225, \infty)$

Table 2: Right-sided 95% confidence intervals for the Pareto tail index of wet and dry epoch durations.

Scale of spatial averaging	2 km	4 km	8 km	16 km	30 km	60 km	120 km
Wet durations sample size (ℓ)	2824	2897	2983	3179	3038	1421	25
Tail-index $m_0 = m_0(\lambda)$	1.62	1.67	1.73	1.79	1.85	1.92	1.99
$J_1(0.05)$, $\delta = 0.05$	*(0, 2.071)	*(0, 1.809)	*(0, 1.899)	*(0, 1.853)	(0, 1.786)	(0, 1.547)	*(0, 4.596)
$J_1(0.05)$, $\delta = 0.25$	*(0, 2.193)	*(0, 2.103)	*(0, 2.064)	*(0, 1.982)	*(0, 2.006)	(0, 1.624)	*(0, 4.918)
$J_1(0.05)$, $\delta = 0.45$	*(0, 2.948)	*(0, 2.639)	*(0, 2.776)	*(0, 2.608)	*(0, 2.558)	*(0, 2.101)	*(0, 75.391)
$I_1(0.05)$, $k =$ plug-in k^*	(0, 1.310)	*(0, 1.834)	*(0, 2.822)	*(0, 10.497)	*(0, 4.023)	*(0, 3.668)	(0, 1.434)
$I_1(0.05)$, $k = \ell/10$	*(0, 1.873)	*(0, 1.876)	*(0, 1.972)	*(0, 1.988)	*(0, 2.374)	*(0, 2.084)	*(0, 2.681)
$I_1(0.05)$, $k = 2\sqrt{\ell}$	*(0, 2.390)	*(0, 2.297)	*(0, 2.477)	*(0, 2.430)	*(0, 3.162)	*(0, 3.267)	(0, 1.235)
Dry durations sample size (ℓ)	2530	2618	2789	3087	3145	1679	45
Tail-index $m_0 = m_0^*(\lambda)$	1.77	1.79	1.83	1.92	2.10	2.62	5.23
$J_1(0.05)$, $\delta = 0.05$	(0, 1.735)	(0, 1.752)	(0, 1.828)	(0, 1.874)	(0, 2.041)	(0, 2.187)	(0, 3.283)
$J_1(0.05)$, $\delta = 0.25$	(0, 1.469)	(0, 1.494)	(0, 1.571)	(0, 1.590)	(0, 1.769)	(0, 1.862)	(0, 2.958)
$J_1(0.05)$, $\delta = 0.45$	(0, 1.322)	(0, 1.309)	(0, 1.359)	(0, 1.377)	(0, 1.500)	(0, 1.631)	(0, 2.506)
$I_1(0.05)$, $k =$ plug-in k^*	*(0, 3.512)	*(0, 2.883)	*(0, 3.157)	*(0, 4.961)	*(0, 5.286)	*(0, 3.161)	*(0, 8.614)
$I_1(0.05)$, $k = \ell/10$	*(0, 2.117)	*(0, 1.964)	*(0, 2.174)	*(0, 1.899)	*(0, 2.158)	*(0, 2.963)	*(0, 8.236)
$I_1(0.05)$, $k = 2\sqrt{\ell}$	*(0, 2.761)	*(0, 2.584)	*(0, 3.062)	*(0, 2.881)	*(0, 3.229)	*(0, 3.307)	(0, 3.282)

Table 3: Left-sided 95% confidence intervals for the Pareto tail index of wet and dry epoch durations.

Scale of spatial averaging	2 km	4 km	8 km	16 km	30 km	60 km	120 km
Wet durations sample size (ℓ)	2824	2897	2983	3179	3038	1421	25
Tail-index $m_0 = m_0(\lambda)$	1.62	1.67	1.73	1.79	1.85	1.92	1.99
JP-estimate, $\delta = 0.05$	1.784	**1.770**	**1.740**	**1.685**	**1.704**	**1.497**	**0.822**
JP-estimate, $\delta = 0.25$	**1.694**	1.484	1.566	1.530	1.470	1.270	0.756
JP-estimate, $\delta = 0.45$	1.430	1.387	1.317	1.337	1.256	1.091	0.780
Hill-estimate, $k =$ plug-in k^*	1.269	1.619	2.242	2.775	3.131	2.931	**0.953**
Hill-estimate, $k = \ell/10$	**1.689**	**1.695**	**1.784**	**1.804**	**2.150**	**1.797**	0.709
Hill-estimate, $k = 2\sqrt{\ell}$	2.008	1.931	2.087	2.052	2.666	2.646	0.593
$I_2(0.05)$, $k =$ plug-in k^*	(1.224, 1.318)	*(1.420, 1.882)	(1.800, 2.969)	*(1.479,22.477)	(2.476, 4.256)	(2.365, 3.854)	*(0.680, 1.588)
$I_2(0.05)$, $k = \ell/10$	*(1.513, 1.912)	*(1.520, 1.916)	*(1.602, 2.013)	*(1.625, 2.027)	(1.932, 2.423)	*(1.543, 2.150)	*(0.378, 5.740)
$I_2(0.05)$, $k = 2\sqrt{\ell}$	(1.687, 2.480)	*(1.624, 2.383)	(1.757, 2.569)	*(1.732, 2.519)	(2.246, 3.279)	(2.158, 3.420)	(0.366, 1.558)
Dry durations sample size (ℓ)	2530	2618	2789	3087	3145	1679	45
Tail-index $m_0 = m_0^*(\lambda)$	1.77	1.79	1.83	1.92	2.10	2.62	5.23
JP-estimate, $\delta = 0.05$	**1.709**	**1.719**	**1.755**	**1.795**	**1.860**	**2.057**	**2.703**
JP-estimate, $\delta = 0.25$	1.437	1.444	1.505	1.550	1.673	1.786	2.344
JP-estimate, $\delta = 0.45$	1.239	1.253	1.318	1.341	1.458	1.572	1.986
Hill-estimate, $k =$ plug-in k^*	2.826	2.394	2.689	3.561	3.698	2.569	**4.134**
Hill-estimate, $k = \ell/10$	**1.898**	**1.764**	**1.960**	**1.721**	**1.958**	**2.586**	3.720
Hill-estimate, $k = 2\sqrt{\ell}$	2.307	2.163	2.571	2.431	2.727	2.702	1.785
$I_2(0.05)$, $k =$ plug-in k^*	(2.293, 3.683)	(1.991, 3.000)	(2.285, 3.266)	(2.665, 5.365)	(2.724, 5.759)	*(2.100, 3.308)	*(2.552, 10.872)
$I_2(0.05)$, $k = \ell/10$	*(1.690, 2.165)	*(1.573, 2.008)	*(1.754, 2.221)	*(1.548, 1.937)	*(1.763, 2.201)	*(2.246, 3.049)	*(2.250, 10.731)
$I_2(0.05)$, $k = 2\sqrt{\ell}$	(1.929, 2.870)	(1.811, 2.684)	(2.158, 3.179)	(2.050, 2.987)	(2.301, 3.347)	*(2.219, 3.455)	(1.156, 3.910)

Table 4: Point estimates $M^*(\delta)$ and $H(k)$ for the Pareto tail index of wet and dry epoch durations, along with two-sided 95% confidence intervals based on the Hill estimator.