Bayesian Score Merging for the Order Restricted RC Association Model

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1 Introduction.

- Let $\boldsymbol{y} = (y_{ij})$ be the frequencies and
- $\boldsymbol{\Pi} = (\pi_{ij})$ is the corresponding probability table

of an $I \times J$ contingency table of two *ordinal* variables X and Y with I and J levels respectively.

Saturated log-linear model:

$$\log \pi_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \quad \lambda_{ij}^{XY} \qquad i = 1, \ldots, I, \; j = 1, \ldots, J.$$

$$\Downarrow$$

$$\log \pi_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \boxed{\phi \mu_i \nu_j} \qquad \text{(Goodman, 1985)}$$

Let $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots \mu_I)$ and $\boldsymbol{\nu} = (\nu_1, \nu_2, \ldots \nu_J)$ be the scores assigned to the levels of X (rows) and Y (columns) respectively.

2 Modeling Details

- Usual imposed constraints on the scores' parameters of the RC model:

$$\sum_{i=1}^{I} \mu_i = \sum_{j=1}^{J} \nu_j = 0 \quad \text{and} \quad \sum_{i=1}^{I} \mu_i^2 = \sum_{j=1}^{J} \nu_j^2 = 1.$$

- We focus on the order restricted version of the RC association model.
- X and Y ordinal $\Rightarrow$ natural to assume that the ordinal structure for scores

$$\mu_1 < \mu_2 < \cdots < \mu_I \quad \text{and} \quad \nu_1 < \nu_2 < \cdots < \nu_J$$

- **Which successive scores $(\mu_i,\ \mu_{i+1})$ and $(\nu_j,\ \nu_{j+1})$ are equal?**
- In all models we assume that at least two row and two column scores are different.

Proposed Constraints

- We propose to use an alternative set of constraints:

$$\mu_1 = \mu_{\min} < \mu_I = \mu_{\max} \text{ and } \nu_1 = \nu_{\min} < \nu_J = \nu_{\max}$$

- Row and column scores take values in the intervals $[\mu_{\min}, \mu_{\max}]$ and $[\nu_{\min}, \nu_{\max}]$ respectively.
- Sensible choices:
  - ◇ $\mu_{\min} = \nu_{\min} = -1$ and $\mu_{\max} = \nu_{\max} = 1$ [range similar to the parameters under constraints (2)]
  - ◇ We use: $\mu_{\min} = \nu_{\min} = 0$ and $\mu_{\max} = \nu_{\max} = 1$
    - ∗ simplifies computations
    - ∗ $\phi = \log\left(\frac{\pi_{11}\pi_{IJ}}{\pi_{1J}\pi_{I1}}\right)$
- Posterior distributions of scores under (2) can be obtained by transforming MCMC output of the proposed parametrization.

Model Formulation

$$\log \pi_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \phi \mu_i \nu_j, \ i = 1, \ldots, I, \ j = 1, \ldots, J$$

- We introduce latent binary indicators

$$\boldsymbol{\gamma} = (1, \gamma_2, \ldots, \gamma_I) \ \text{ and } \ \boldsymbol{\delta} = (1, \delta_2, \ldots, \delta_J) \ \text{ and}$$

  which are equal to

  $\gamma_i = 1$ when $\mu_i > \mu_{i-1}$ (or $\delta_j = 1$ when $\nu_j > \nu_{j-1}$)

  $\gamma_i = 0$ when $\mu_i = \mu_{i-1}$ (or $\delta_j = 0$ when $\nu_j = \nu_{j-1}$)

- The vectors $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$:
  - specify which scores are equal
  - are used instead of the usual model indicator m

Let us now define

$$\Gamma_i = \sum_{k=1}^{i} \gamma_k \;\; \text{and} \;\; \Delta_j = \sum_{k=1}^{j} \delta_k$$

are the distinct different scores under estimation until row i or column j respectively.

Moreover the actual distinct unequal row and column scores will be denoted by the vectors $\boldsymbol{\mu}_\gamma$ and $\boldsymbol{\nu}_\delta$ of dimension Γ_I and Δ_J of respectively given by

$$\boldsymbol{\mu}_\gamma = \Big(\{\mu_i : \gamma_i = 1; i = 1, 2, \ldots, I\} \Big) = \Big(\mu_\gamma(1), \mu_\gamma(2), \ldots, \mu_\gamma(\Gamma_I) \Big)^T$$

and

$$\boldsymbol{\nu}_\delta = \Big(\{\nu_j : \delta_j = 1; j = 1, 2, \ldots, J\} \Big) = \Big(\nu_\delta(1), \nu_\delta(2), \ldots, \nu_\delta(\Delta_J) \Big)^T.$$

Then the original scores are given by

$$\mu_i = \mu_\gamma(\Gamma_i) \;\; \text{and} \;\; \nu_j = \nu_\delta(\Delta_j)$$

Prior Distributions on Scores

Equivalently, the scores are a priori distributed as ordered iid uniform random variables

$$f(\boldsymbol{\mu}) = \frac{(\Gamma_I - 2)!}{(\mu_{\max} - \mu_{\min})^{\Gamma_I - 2}} \mathcal{I}(\mu_{\min} < \text{ordered different } \mu\text{'s} < \mu_{\max})$$

Similarly, for the column scores

$$f(\boldsymbol{\nu}) = \frac{(\Delta_J - 2)!}{(\nu_{\max} - \nu_{\min})^{\Delta_J - 2}} \mathcal{I}(\nu_{\min} < \text{ordered different } \nu\text{'s} < \nu_{\max})$$

Prior Distributions on the rest of parameters

Normal with large variances for the rest of the parameters.

Bernoulli for γ_i and δ_j with prior probabilities equal to 1/2.

3 RJMCMC algorithm

1. Update model structure: Sample $(\boldsymbol{\gamma}, \boldsymbol{\delta})$ using successive RJMCMC moves:
   - For $i = 2, \ldots, I$, propose $\boldsymbol{\gamma}'$: $\gamma'_i = 1 - \gamma_i$, $\gamma'_k = \gamma_k$ for $k \neq i$.
     - **Split**: if $(\gamma_i = 0) \rightarrow (\gamma'_i = 1)$ then propose $(\mu_{i-1} = \mu_i) \rightarrow (\mu'_{i-1} < \mu'_i)$.
       (a) Generate u from $q(u|\boldsymbol{\mu}, \boldsymbol{\gamma}, \boldsymbol{\gamma}')$.
       (b) Set $\boldsymbol{\mu}'_{\gamma'} = g(\boldsymbol{\mu}_{\gamma}, u)$.
       (c) Obtain $\boldsymbol{\mu}'$ from $\boldsymbol{\mu}'_{\gamma'}$ via $\mu_i = \mu_{\gamma}(\mathit{\Gamma}_i)$.
     - **Merge**: if $(\gamma_i = 1) \rightarrow (\gamma'_i = 0)$ then propose $(\mu_{i-1} < \mu_i) \rightarrow (\mu'_{i-1} = \mu'_i)$.
       (a) Set $(\boldsymbol{\mu}'_{\gamma'}, u) = g^{-1}(\boldsymbol{\mu}_{\gamma})$.
       (b) Obtain $\boldsymbol{\mu}'$ from $\boldsymbol{\mu}'_{\gamma'}$ via $\mu_i = \mu_{\gamma}(\mathit{\Gamma}_i)$.
   - Similar is scheme for updating the components of $\boldsymbol{\delta}$.
2. Generate model parameters $(\boldsymbol{\lambda}^X, \boldsymbol{\lambda}^Y, \phi, \boldsymbol{\mu}, \boldsymbol{\nu})$, given the model structure $(\boldsymbol{\gamma}, \boldsymbol{\delta})$:
   - Sample row and column effects.
   - Sample ϕ using a simple random walk Metropolis.
   - Use random walk on logits of column and row scores' differences.

The probability of acceptance of the proposed move $(\boldsymbol{\gamma}, \boldsymbol{\mu}) \rightarrow (\boldsymbol{\gamma}', \boldsymbol{\mu}')$ in each RJMCMC step equals $\alpha = \min(1, A)$, where

$$A = \frac{f(y|\boldsymbol{\lambda}^X, \boldsymbol{\lambda}^Y, \phi, \boldsymbol{\mu}', \boldsymbol{\nu})}{f(y|\boldsymbol{\lambda}^X, \boldsymbol{\lambda}^Y, \phi, \boldsymbol{\mu}, \boldsymbol{\nu})} \frac{f(\boldsymbol{\mu}'_{\gamma'}|\boldsymbol{\gamma}')f(\boldsymbol{\gamma}')}{f(\boldsymbol{\mu}_{\gamma}|\boldsymbol{\gamma})f(\boldsymbol{\gamma})} \frac{q(u|\boldsymbol{\mu}'_{\gamma'}, \boldsymbol{\gamma}', \boldsymbol{\gamma})^{\gamma_i}}{q(u|\boldsymbol{\mu}_{\gamma}, \boldsymbol{\gamma}, \boldsymbol{\gamma}')^{1-\gamma_i}} |J|^{1-2\gamma_i},$$

$|J|$ is the absolute value of the RJMCMC Jacobian used in the split move and is given by

$$|J| = \left| \frac{\partial g(\boldsymbol{\mu}_{\gamma}, u)}{\partial(\boldsymbol{\mu}_{\gamma}, u)} \right| .$$

Merge Central Scores
$(\gamma_i = 1 \rightarrow \gamma'_i = 0,\ \ i : 2 < \Gamma_i = \ell < \Gamma_I)$

$$\Big(\ldots \leq \mu_\gamma(\ell-2) < \qquad \underbrace{\mu_\gamma(\ell-1) \quad {\color{red}<} \quad \mu_\gamma(\ell)} \qquad < \mu_\gamma(\ell+1) \leq \ldots\Big)$$

$$\Downarrow \qquad\qquad\qquad \Downarrow \qquad\qquad\qquad \Downarrow$$

$$\Big(\ldots \leq \mu'_{\gamma'}(\ell-2) < \qquad\qquad \mu'_{\gamma'}(\ell-1) \qquad\qquad < \mu'_{\gamma'}(\ell) \leq \ldots\Big)$$

$$\Downarrow$$

Usual transformation: $\mu'_{\gamma'}(\ell-1) = \frac{\mu_\gamma(\ell-1)+\mu_\gamma(\ell)}{2}$

and leave the rest of the scores unchanged

$$\mu'_{\gamma'}(k) = \begin{cases} \mu_\gamma(k) & \text{for } k \ < \ \ell-1 \\ \mu_\gamma(k+1) & \text{for } k \ > \ \ell-1 \end{cases}$$

Split Central Scores (inverse move)
$(\gamma_i = 0 \rightarrow \gamma'_i = 1,\ \ i : 2 \leq \Gamma_i = \ell < \Gamma_I)$

$$\Big(\ldots \leq \mu_\gamma(\ell - 1) < \qquad \mu_\gamma(\ell) \qquad < \mu_\gamma(\ell+1) \leq \ldots \Big)$$

$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

$$\Big(\ldots \leq \mu'_{\gamma'}(\ell - 1) < \overbrace{\mu'_{\gamma'}(\ell) \quad {\color{red}<} \quad \mu'_{\gamma'}(\ell+1)} < \mu'_{\gamma'}(\ell+2) \leq \ldots \Big)$$

$$\downarrow \qquad\qquad \downarrow$$

$$\mu_\gamma(\ell) - u \qquad \mu_\gamma(\ell) + u$$

- Generate $u \in \Big(0, \min\big\{\ \mu_\gamma(\ell) - \mu_\gamma(\ell-1),\ \ \mu_\gamma(\ell+1) - \mu_\gamma(\ell)\ \big\}\Big)$
- Set $\mu'_{\gamma'}(\ell) = \mu_\gamma(\ell) - u$ and $\mu'_{\gamma'}(\ell+1) = \mu_\gamma(\ell) + u$.

- Leave the rest of the scores unchanged, i.e. set

$$\mu'_{\gamma'}(k) = \begin{cases} \mu_\gamma(k) & \text{for } k < \ell \\ \mu_\gamma(k-1) & \text{for } k > \ell+1 \end{cases}$$

From the above we have

- $|J| = 2$ and $u = \frac{\mu'_{\gamma'}(\ell+1) - \mu'_{\gamma'}(\ell)}{2}$
- Hence in MERGE MOVE $\rightarrow |J| = \dfrac{1}{2}$ and $u = \dfrac{1}{2}\Big\{\mu_\gamma(\ell) - \mu_\gamma(\ell-1)\Big\}$.

PROBLEM

The above transformation cannot be applied for merging/spliting the **lowest** or the **highest** scores.

Merge the Lowest Scores $\mu_\gamma(1)$ and $\mu_\gamma(2)$
$(\gamma_i = 1 \rightarrow \gamma'_i = 0,\ \ i : \Gamma_i = 2)$

$$\underbrace{\mu_{\min} = \mu_\gamma(1) \ < \ \mu_\gamma(2)} < \quad \mu_\gamma(3) \quad < \ldots$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\mu_{\min} = \mu'_{\gamma'}(1) \qquad < \qquad \mu'_{\gamma'}(2) \quad < \ldots$$

$$\Downarrow \qquad\qquad \Downarrow$$

Usual Transformation $\quad \frac{\mu_{\min}+\mu_\gamma(2)}{2} < \qquad \mu_\gamma(3) \quad < \ldots$

Not Valid Since $\quad \neq \mu_{\min}$

(VIOLATES THE CONSTRAINT $\mu'_{\gamma'}(1) = \mu_{\min}$)

Using similar logic we apply the following transformations

$$
\begin{array}{ccccc}
\underbrace{\mu_{\min} = \mu_\gamma(1) < \mu_\gamma(2)} & < & \mu_\gamma(3) & < \ldots < & \mu_\gamma(\mathit{\Gamma}_I) = \mu_{\max} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\frac{\mu_{\min}+\mu_\gamma(2)}{2} & < & \mu_\gamma(3) & < \ldots < & \mu_\gamma(\mathit{\Gamma}_I) = \mu_{\max} \\
\Downarrow & & \Downarrow & & \Downarrow \\
0 & < & \mu_\gamma(3) - \frac{\mu_{\min}+\mu_\gamma(2)}{2} & < \ldots < & \mu_{\max} - \frac{\mu_{\min}+\mu_\gamma(2)}{2} \\
\Downarrow & & \Downarrow & & \Downarrow \\
0 & < & \frac{\mu_\gamma(3)-\frac{\mu_{\min}+\mu_\gamma(2)}{2}}{\mu_{\max}-\frac{\mu_{\min}+\mu_\gamma(2)}{2}} & < \ldots < & 1 \\
\Downarrow & & \Downarrow & & \Downarrow \\
\mu_{\min} & < & \mu_{\min} + \frac{2\mu_\gamma(3)-\mu_{\min}-\mu_\gamma(2)}{2\mu_{\max}-\mu_{\min}-\mu_\gamma(2)}(\mu_{\max}-\mu_{\min}) & < \ldots < & \mu_{\max} \\
\downarrow & & \downarrow & & \downarrow \\
\mu'_{\gamma'}(1) & < & \mu'_{\gamma'}(2) & < \ldots < & \mu'_{\gamma'}(\mathit{\Gamma}'_I)
\end{array}
$$

Merge the Lowest Scores $\mu_\gamma(1)$ and $\mu_\gamma(2)$ $(\gamma_i = 1 \to \gamma'_i = 0,\ \ i : \mathit{\Gamma}_i = 2)$

Final transformation

$$\mu'_{\gamma'}(k) = \begin{cases} \mu_{\min}, & k = 1, \\ \mu_{\min} + (\mu_{\max} - \mu_{\min}) \dfrac{2\mu_\gamma(k+1) - \mu_{\min} - \mu_\gamma(2)}{2\mu_{\max} - \mu_{\min} - \mu_\gamma(2)}, & k > 1. \end{cases} \quad (2)$$

Split the Lowest Score $\mu_\gamma(1)$ (reverse move) $(\gamma_i = 0 \rightarrow \gamma'_i = 1,\ \ i : \Gamma_i = 1)$

$$\Big(\mu_{\min} = \mu_\gamma(1) \quad < \quad \mu_\gamma(2) \quad < \dots\Big)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\Big(\overbrace{\mu_{\min} = \mu'_{\gamma'}(1) \;\; {\color{red}<} \;\; \mu'_{\gamma'}(2)} < \quad \mu'_{\gamma'}(3) \quad < \dots\Big)$$

Transformation

$$\mu'_{\gamma'}(k) = \begin{cases} \mu_{\min}, & k = 1, \\ u, & k = 2, \\ \dfrac{1}{2}\left\{\mu_{\min} + u + (2\mu_{\max} - \mu_{\min} - u)\dfrac{\mu_\gamma(k-1) - \mu_{\min}}{\mu_{\max} - \mu_{\min}}\right\}, & k > 2. \end{cases} \qquad (3)$$

- In Split Move → Generate u in the interval

$$u \in \left(\mu_{\min},\ \mu_\gamma(2) + \frac{(\mu_\gamma(2) - \mu_{min})[\mu_{max} - \mu_\gamma(2)]}{\mu_\gamma(2) + \mu_{max} - 2\mu_{min}} \right)$$

- Calculate $|J| = \left(1 - \frac{1}{2}\frac{u-\mu_{\min}}{\mu_{\max}-\mu_{\min}}\right)^{\Gamma_I - 2}$
-
- In Megre Move → $u = \mu_\gamma(2)$ and

$$|J| = \left[\left(1 - \frac{1}{2}\frac{u - \mu_{\min}}{\mu_{\max} - \mu_{\min}}\right)^{\Gamma_I' - 2}\right]^{-1} = \left(1 - \frac{1}{2}\frac{\mu_\gamma(2) - \mu_{\min}}{\mu_{\max} - \mu_{\min}}\right)^{3 - \Gamma_I} .$$

Reminder:

- Γ_I is the number of scores of the current model (In split “smaller”, In merge: “larger” model)
- Γ_I' is the number of scores of the proposed model (In split “larger”, In merge: “smaller” model)

Merge the Highest Scores $\mu_\gamma(\Gamma_I - 1)$ and $\mu_\gamma(\Gamma_I)$ $(\gamma_i = 1 \rightarrow \gamma_i' = 0,\ i : \Gamma_i = \Gamma_I)$

$$\mu_{\min} = \mu_\gamma(1) \quad < \ldots < \quad \mu_\gamma(\Gamma_I - 2) < \quad \underbrace{\mu_\gamma(\Gamma_I - 1) \; < \; \mu_\gamma(\Gamma_I) = \mu_{\max}}$$

$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

$$\mu_{\min} = \mu'_{\gamma'}(1) \quad < \ldots < \quad \mu'_{\gamma'}(\Gamma_I - 2) < \qquad \mu'_{\gamma'}(\Gamma_I - 1) = \mu_{\max}$$

Final transformation

$$\mu'_{\gamma'}(k) = \begin{cases} \mu_{\min} + 2(\mu_{\max} - \mu_{\min}) \dfrac{\mu_\gamma(k) - \mu_{\min}}{\mu_\gamma(\Gamma_I - 1) + \mu_{\max} - 2\mu_{\min}}, & k \leqslant \Gamma_I' - 1 = \Gamma_I - 2, \\ \mu_{\max}, & k = \Gamma_I' = \Gamma_I - 1. \end{cases} \tag{4}$$

Note: $\Gamma_I' = \Gamma_I - 1$ since we merge two scores into one.

Split the Highest Score $\mu_\gamma(\Gamma_I)$ (reverse move) $(\gamma_i = 0 \rightarrow \gamma'_i = 1,\ i : \Gamma_i = \Gamma_I\ )$

$$\mu_{\min} = \mu_\gamma(1) \quad < \ldots < \quad \mu_\gamma(\Gamma_I - 1) < \qquad\qquad \mu_\gamma(\Gamma_I) = \mu_{\max}$$

$$\Downarrow \qquad\qquad\qquad \Downarrow \qquad\qquad\qquad \Downarrow$$

$$\mu_{\min} = \mu'_{\gamma'}(1) \quad < \ldots < \quad \mu'_{\gamma'}(\Gamma_I - 1) < \quad \overbrace{\mu'_{\gamma'}(\Gamma_I) \ {\color{red}<}\ \mu'_{\gamma'}(\Gamma_I + 1) = \mu_{\max}}$$

Final transformation

$$\mu'_{\gamma'}(k) = \begin{cases} \mu_\gamma(k) - \frac{u}{2}\frac{\mu_\gamma(k) - \mu_{\min}}{\mu_{\max} - \mu_{\min}}, & k \leqslant \Gamma'_I - 2 = \Gamma_I - 1 \\ \mu_{\max} - u, & k = \Gamma'_I - 1 = \Gamma_I \\ \mu_{\max}, & k = \Gamma'_I = \Gamma_I + 1. \end{cases} \tag{5}$$

- **In Split move**
  - Generate u in the interval

$$u \in \left(0, 2\frac{\Big(\mu_{\max} - \mu_{\min}\Big)\Big(\mu_{\max} - \mu_{\gamma}(\Gamma_I - 1)\Big)}{\Big(\mu_{\max} - \mu_{\min}\Big) + \Big(\mu_{\max} - \mu_{\gamma}(\Gamma_I - 1)\Big)}\right)$$

  - Determinant of the Jacobian: $|J| = \left(1 - \frac{1}{2}\frac{u}{\mu_{\max} - \mu_{\min}}\right)^{\Gamma_I - 2}$
  - Γ_I is the number of scores in the smaller (current) model.

- **In Merge move**
  - $u = \mu_{\max} - \mu_{\gamma}(\Gamma_I - 1)$ and
  - Det. of Jacobian:
    $|J| = \left(1 - \frac{1}{2}\frac{u}{\mu_{\max} - \mu_{\min}}\right)^{2-\Gamma'_I} = \left(1 - \frac{1}{2}\frac{\mu_{\max} - \mu_{\gamma}(\Gamma_I - 1)}{\mu_{\max} - \mu_{\min}}\right)^{3-\Gamma_I}$
  - Here:
    * Γ_I is the number of scores in the "bigger" (current) model.
    * Γ'_I is the number of scores in the "smaller" (proposed) model.

Additional Details

- In practice we have used $\mu_{\min} = \nu_{\min} = 0$ and $\mu_{\max} = \nu_{\max} = 1$.
- When $\mathit{\Gamma}_I = 1$ then two scores are different and set equal to $\mu_{\min}$ and $\mu_{\max}$. No further splitting is allowed. Similar is the case for column scores ν_j.
- Rescaled Beta proposals can be used for u.
- In practice we have used **Uniform** proposal which proved sufficient for two dataset we have implemented the methodology.
- Further investigation is needed in order to construct proposals leading to more efficient RJMCMC schemes.

4 Illustrative Example.

Classical dataset of Maxwell (1961) concerning the severity of dreams' disturbance of 223 boys aged from 5 to 15 years.

	Disturbance (from low to high)				
Age Group	1	2	3	4	Total
5– 7	7	4	3	7	21
8– 9	10	15	11	13	49
10–11	23	9	11	7	50
12–13	28	9	12	10	59
14–15	32	5	4	3	44
Total	100	42	41	40	223

Results: Most frequently visited models

k	**Model (scores)**	**Post. prob.**	PO_{1k}	**AIC**	**BIC**	**DIC**	p_m	d_m
1	$\mu_1 = \mu_2 < \mu_3 = \mu_4 < \mu_5$ $\nu_1 < \nu_2 = \nu_3 = \nu_4$	0.1620	1.00	1265.0	1295.7	1265.0	9.0	9
2	$\mu_1 = \mu_2 < \mu_3 = \mu_4 < \mu_5$ $\nu_1 < \nu_2 = \nu_3 < \nu_4$	0.1540	1.05	1265.9	1300.0	1265.1	9.6	10
3	$\mu_1 = \mu_2 < \mu_3 < \mu_4 < \mu_5$ $\nu_1 < \nu_2 = \nu_3 = \nu_4$	0.0877	1.85	1267.6	1301.6	1266.3	9.4	10
4	$\mu_1 = \mu_2 < \mu_3 < \mu_4 < \mu_5$ $\nu_1 < \nu_2 = \nu_3 < \nu_4$	0.0725	2.23	1268.6	1306.1	1266.4	9.9	11
5	$\mu_1 = \mu_2 < \mu_3 = \mu_4 < \mu_5$ $\nu_1 < \nu_2 < \nu_3 < \nu_4$	0.0609	2.66	1269.0	1306.5	1266.4	9.7	11
6	$\mu_1 = \mu_2 < \mu_3 = \mu_4 < \mu_5$ $\nu_1 < \nu_2 < \nu_3 = \nu_4$	0.0579	2.80	1267.6	1301.7	1266.5	9.4	10
7	$\mu_1 < \mu_2 < \mu_3 = \mu_4 < \mu_5$ $\nu_1 < \nu_2 = \nu_3 < \nu_4$	0.0541	2.99	1269.0	1306.5	1266.7	9.9	11
8	$\mu_1 < \mu_2 < \mu_3 = \mu_4 < \mu_5$ $\nu_1 < \nu_2 = \nu_3 = \nu_4$	0.0522	3.10	1268.3	1302.4	1266.8	9.2	10

Single RJMCMC (R RESULTS): 100,000 iterations + additional burn-in of 10,000 iterations.

Results: Marginal Probabilities $f(\gamma_i = 1|\boldsymbol{y})$ and $f(\delta_j = 1|\boldsymbol{y})$

Row Scores	Posterior Probability	Column Scores	Posterior Probability
$f(\gamma_2 = 1\|\boldsymbol{y}) =$	0.285	$f(\delta_2 = 1\|\boldsymbol{y}) =$	0.996
$f(\gamma_3 = 1\|\boldsymbol{y}) =$	0.940	$f(\delta_3 = 1\|\boldsymbol{y}) =$	0.286
$f(\gamma_4 = 1\|\boldsymbol{y}) =$	0.391	$f(\delta_4 = 1\|\boldsymbol{y}) =$	0.484
$f(\gamma_5 = 1\|\boldsymbol{y}) =$	0.964		

Single RJMCMC (R RESULTS): 100,000 iterations + additional burn-in of 10,000 iterations.

Some Comments on the Results

- Negative association between age and severity of dreams' distrurbance ($\phi < 0$).
- **Age**: the first two categories as well as the third and the fourth are indistinguishable in terms of the association for the severity of dreams' distrurbance (marginal posterior probabilities = 0.71 and 0.63 respectively).
- **Severity of dreams' distrurbance**: More uncertainty is involved in their categories:
  - ◇ It is clear that the first one differs than the rest [$f(\delta_2 = 1|\boldsymbol{y}) = 0.996$].
  - ◇ Model with the highest posterior probability ⇒ all the other three scores equal ($\nu_2 = \nu_3 = \nu_4$).
  - ◇ Model with the 2nd highest posterior probability ⇒ $\nu_2 = \nu_3 < \nu_4$.
- The algorithm was highly mobile visiting 69, 86 and all 105 models in 10, 100 iterations 400 thousand iterations respectively.
- RJMCMC indicated a more parsimonious model (according to highest posterior probability) than the one (2nd in rank) indicated by our previous analysis (see Iliopoulos *et al*. 2006).

5 Work in progress and future work

1. Incorporate selection between order restricted Row and Column association models
2. Comparison of the above models with the Uniform association, Independence and Saturated models [use different prior for ϕ].
3. Incorporate selection between unrestricted RC, Row, Column association models (can we use similar parametrization?)
4. Use similar approach in unrestricted RC model for merging/grouping scores
5. Expand methodology to high dimensional tables
6. Use different priors for scores; for example power prior and imaginary data.

Other Publications by the same Group

Kateri, M., Nicolaou, A. and Ntzoufras, I. (2005). Bayesian Inference for the RC(m) Association Model. *Journal of Computational and Graphical Statistics*, **14**, 116–138.

Iliopoulos, G., Kateri, M. and Ntzoufras, I. (2006). Bayesian Estimation of Unrestricted and Order-Restricted Association Models for a Two-Way Contingency Table (to appear). *Computational Statistics and Data Analysis*.

Iliopoulos, G., Kateri, M. and Ntzoufras, I. (2007). Bayesian Model Comparison for the Order Restricted RC Association Model (in progress).

Related Work

Tarantola, C., Consonni, G. and Dellaportas, P. (2007) Bayesian clustering for row effects models. *Technical Report*, University of Pavia.

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Goodman, L.A. (1979). Simple models for the analysis of association in cross-classifications having ordered categories. *Journal of the American Statistical Association*, **74**, 537–552.

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Maxwell, A.E. (1961). *Analyzing Qualitative Data.* London: Methuen.