Chapter 4

Non-Normality in Control Charts

4.1 Introduction

In control charting methodology an assumption often used to determine statistical properties is that the data are normally distributed. However, it can be shown that this assumption is critical for the performance of the control charts. In Section 4.2, we present the non-normality effect in Univariate and Multivariate Shewhart Charts. In Section 4.3 the ascription under non-normality in univariate and multivariate EWMA Charts is given. The EWMA control charts for dispersion are investigated in detail and results on their performance are given together with some recommendations.

4.2 Non-Normality in Univariate and Multivariate Shewhart Charts

The usual way of constructing the Shewhart charts is by assuming normality for the underlying characteristic. In the case of nonnormality if we know the exact distribution of the characteristic plotted we may construct the corresponding probability limits without a problem. The case that appears to be the most difficult is when we do not have a normally distributed characteristic and the probability density function of this characteristic is not known. Then, we have two alternatives; either use nonparametric control charts (see Chakraborti et al. (2001)) or use the existing theory developed for a normally distributed variable. For this second case, Burr (1967) examined the effect of nonnormality on the often used constants in the Shewhart control charts and concluded that they are robust to the assumption of normality except in cases of extremely non-normal distributions. Additionally, Schilling and Nelson (1976) surveyed on the effect of non-normality on the control limits of the $\overline{X}$ chart. They found that usually a sample of size 5 is enough to ensure the robustness to normality of the control limits. Yourstone and Zimmer (1992) proposed the use of the generalized Burr distribution for determining non-symmetrical limits for a control chart for sample averages. They focused on the effect of non-normality measured by the skewness and kurtosis on the ARL values. They concluded that a large skewness or kurtosis in the original data will result in sizeable large skewness or kurtosis values for the sample averages. Therefore, the practitioner should consider non-symmetrical control charts. Janacek and Meikle (1997) proposed the use of control charts of medians in the case of non-normal data. They assumed that at the beginning the process is in-control and we collect a reference sample of size N. Then, we take samples of size n to check if the process remains in-control in terms of location. Let B be the number of members of the test sample less than k_q where $F_X(k_q) = q$. If the distribution of the reference sample $F_X(x)$ is unknown and $\widehat{m}$ is the sample median, then

$$P\left(B=b\right)=\frac{\binom{j+b-1}{b}\binom{N+n-j-b}{n-b}}{\binom{N+n}{n}}.$$

It can be proved that

$$P\left(x_{(j)}<\widehat{m}<x_{(N-j+1)}\right)=1-2\sum_{b=[n/2]+1}^{n}\frac{\binom{j+b-1}{b}\binom{N+n-j-b}{n-b}}{\binom{N+n}{n}}$$

and this relationship can be used to construct suitable control limits. As Janacek and Meikle (1997) indicate their proposed approach is very reliable when we have non-normal data, but when used with normal data there is a loss of power.

The nonnormality effect on the T^2 control charts have been studied by many authors such as Chase and Bulgren (1971), Mardia (1974, 1975), Everitt (1979), Bauer (1981), Tiku and Singh (1982) and Srivastava and Awan (1982). They proved through simulation that this statistic is affected by nonnormal distributions and especially in the case of the highly skewed ones.

4.3 Non-Normality in Univariate and Multivariate EWMA Charts

The assumption of normality in the EWMA chart has drawn the attention of researchers in the last years. In subsection 4.3.1 we present the recent results on this field for the EWMA chart for the mean in univariate and multivariate cases. Moreover, some new results (Maravelakis et al. (2003)) about the robustness to normality of the EWMA charts for dispersion are given in subsections 4.3.2-4.3.5.

4.3.1 The EWMA control charts for monitoring the process mean

The EWMA is a popular chart for detecting small to moderate shifts and because of another characteristic. As Montgomery (2001) states "It is almost a perfectly non-parametric (distribution free) procedure". Borror et al. (1999), examined the ARL performance of the EWMA chart for the mean in non-normal cases when the parameters of the process are known and concluded in the same result for certain values of the smoothing parameter. They proposed that an EWMA chart with smoothing parameter equal to 0.05 is very effective in the case of nonnormality. Its in-control value is very close to the one for the normal case. Furthermore, it does not lose its ability to detect fast an out-of-control situation. However, as the value of the smoothing parameter increases the performance of the chart under nonnormality is not that good. Recently, Stoumbos and Sullivan (2002) and Testik et al. (2003) extended the work of Borror et al. (1999) to the multivariate case of the EWMA chart. They concluded that a properly designed multivariate EWMA control chart is robust to the non-normality assumption. In particular, Stoumbos and Sullivan (2002) showed that for up to five dimensions a value of the smoothing parameter in the range $[0.02, 0.05]$ is enough to preserve performance as in the multinormality case. However, when we have more than five dimensions a value of 0.02 or less is needed for the MEWMA chart to behave as under multinormality.

4.3.2 The EWMA control charts for monitoring the process dispersion

Let μ_0 and σ_0 denote the in-control values of the process parameters that are either known or estimated from a very large sample taken when the process is assumed to be in-control. We want to detect any shifts of the dispersion in the process using EWMA charts that are known to be efficient for detecting small to moderate shifts in the parameters. For the remaining of this study we assume that we have independent and identically distributed data with sample size unity and also that we are in the prospective setting (Phase II) where the estimates or the parameter values are used to monitor the process. In the case of rational subgroups the central limit theorem applies and therefore the non normality issue does not bother us as much.

Several publications dealing with the subject of detecting shifts in the dispersion using an EWMA type chart have appeared in the literature (see, e.g. Domangue and Patch (1991), MacGregor and Harris (1993), Acosta-Mejia and Pignatiello (2000)). Our main concern is to detect increases in the process dispersion. We have to stress though, that detecting decreases in the dispersion is equally important because they indicate an improvement in the process. Nevertheless, it is not probable that a reduction in the process standard deviation, or variance, will occur without a corrective action. Therefore, when an attempt to improve the quality of a process is taking place, the time that this possible change occurs is known. A control chart is one of the tools to check for possible reduction in the variance before and after the corrective action. However, the main use of a control chart is to detect persistent or sudden shifts in a process at unknown times.

Table 4.1 In-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.05$

		WR	SR	HO	DP1	DP2
	h	2.876	2.604	2.436	2.1492	2.495
N(μ,σ^2)	ARL	370.4	370.4	370.4	370.4	370.4
	MRL	260	260	264	259	257
	SDRL	361.3	358.1	353.6	361.8	368.3
G(4,1)	ARL	151.3	304.2	444.1	490.5	181.2
	MRL	106	213	312	340	124
	SDRL	148.0	296.3	431.7	486.5	183.0
G(3,1)	ARL	133.1	290.6	473.2	535.2	162.6
	MRL	93	205	331	372	111
	SDRL	131.0	283.0	461.9	532.6	166.0
G(2,1)	ARL	112.4	267.5	522.5	641.5	140.3
	MRL	79	187	365	444	95
	SDRL	110.0	262.1	511.3	640.5	144.1
G(1,1)	ARL	84.1	225.3	659.4	1048.1	111.8
	MRL	59	158	461	723	75
	SDRL	82.7	220.7	647.9	1056.6	116.1
G(0.5,1)	ARL	67.8	185.8	840.3	2449.9	94.8
	MRL	47	130	583	1679	63
	SDRL	66.8	184.3	837.1	2489.1	99.9

The EWMA chart of squared deviations from target ($\mathrm{EWMA_S}$) was proposed by Wortham and Ringer (1971) for detecting a shift in the process standard deviation. The statistic of this chart is given by

$$S_i = \lambda(x_i - \mu_0)^2 + (1-\lambda)\max(S_{i-1}, \sigma_0^2),\ S_0 = \sigma_0^2,$$

Table 4.1 (continued) In-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.1$

		WR	SR	HO	DP1	DP2
	h	3.432	2.916	2.628	2.409	3.094
N(μ,σ^2)	ARL	370.4	370.4	370.4	370.4	370.4
	MRL	259	257	260	259	258
	SDRL	365.9	360.8	359.2	363.6	367.4
G(4,1)	ARL	129.7	237.0	380.8	421.1	147.2
	MRL	91	166	265	293	102
	SDRL	127.7	231.7	374.1	418.8	147.3
G(3,1)	ARL	114.3	218.0	382.1	437.2	130.7
	MRL	79	152	267	304	90
	SDRL	113.2	214.5	373.8	433.7	131.3
G(2,1)	ARL	95.6	191.6	388.3	472.1	111.8
	MRL	66	133	271	328	77
	SDRL	94.8	188.9	382.1	469.3	112.7
G(1,1)	ARL	72.5	150.6	393.3	569.5	87.0
	MRL	51	105	273	396	60
	SDRL	71.2	148.3	388.3	570.5	88.2
G(0.5,1)	ARL	59.2	120.2	399.4	816.4	73.1
	MRL	41	83	278	564	50
	SDRL	58.6	119.1	395.1	822.3	74.7

where λ is a smoothing parameter that takes values between 0 and 1 and S_0 is the initial value. The above statistic is one-sided and it is defined in a way to detect only upward shifts. This happens because, whenever S_i is less than σ_0^2, we set it equal to its starting value. The control limit of this chart is

$$UCL = \sigma_0^2 + h_S \sigma_0^2 \sqrt{\left(\frac{2\lambda}{2-\lambda}\right)},$$

Table 4.1 (continued) In-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.2$

		WR	SR	HO	DP1	DP2
	h	4.112	3.215	2.742	2.584	3.821
N(μ,σ^2)	ARL	370.4	370.4	370.4	370.4	370.4
	MRL	256	257	257	259	258
	SDRL	368.9	363.4	363.3	366.4	368.8
G(4,1)	ARL	113.4	171.8	281.2	319.7	121.9
	MRL	79	120	196	221	84
	SDRL	112.6	169.0	277.9	318.3	121.4
G(3,1)	ARL	99.7	154.3	263.7	310.5	107.5
	MRL	69	107	184	216	75
	SDRL	98.9	153.1	260.7	308.2	107.5
G(2,1)	ARL	83.5	131.4	240.8	296.4	91.3
	MRL	58	92	167	205	63
	SDRL	82.7	129.5	238.0	294.4	91.3
G(1,1)	ARL	64.1	100.6	205.5	279.1	70.7
	MRL	45	70	144	194	49
	SDRL	63.3	99.3	202.6	277.7	70.9
G(0.5,1)	ARL	52.5	81.0	179.6	291.4	59.4
	MRL	36	57	125	201	41
	SDRL	51.8	80.3	178.3	293.2	59.4

where h_S is a constant used to specify the width of the control limit. Note that σ_0^2 would be the mean and $\sigma_0^2\sqrt{2\lambda/(2-\lambda)}$ would be the asymptotic standard deviation of S_i if the reset was not used. However, the control limit is not modified in order to resemble the form of an asymptotic EWMA control limit (Reynolds and Stoumbos (2001)).

As Stoumbos and Reynolds (2000) indicate, when the normality assumption is questionable for the observations, the $\mathrm{EWMA_S}$ statistic does not converge quickly to normality

Table 4.2. Out-of-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.05$

Shift		1.2					1.4				
		WR	SR	HO	DP1	DP2	WR	SR	HO	DP1	DP2
$N(\mu, \sigma^2)$	ARL	113.3	116.2	126.1	114.4	100.8	55.6	58.4	65.9	58.6	48.8
	MRL	81	84	92	82	72	41	44	50	44	37
	SDRL	105.5	105.1	113.1	104.5	94.2	48.2	48.5	54.2	48.7	42.5
G(4,1)	ARL	66.0	111.0	167.3	171.9	68.8	36.2	54.5	80.2	77.6	35.5
	MRL	47	80	120	121	48	27	40	59	56	26
	SDRL	62.6	103.0	155.7	164.8	67.5	32.9	47.8	70.2	70.0	33.3
G(3,1)	ARL	64.4	115.4	185.2	193.6	68.9	37.9	59.9	92.0	91.5	37.9
	MRL	46	82	132	136	48	27	44	67	66	27
	SDRL	61.7	108.2	174.0	187.4	68.1	35.1	53.8	82.1	84.7	36.1
G(2,1)	ARL	61.1	119.3	214.1	237.9	67.6	39.2	67.0	111.9	116.8	40.6
	MRL	43	85	152	166	46	28	48	81	83	28
	SDRL	58.6	113.4	203.1	233.4	68.0	36.8	61.2	101.9	110.5	39.5
G(1,1)	ARL	54.6	121.3	294.0	393.6	64.8	39.1	76.8	164.0	196.6	43.3
	MRL	39	86	206	271	44	28	55	117	137	29
	SDRL	52.7	116.5	284.7	395.9	66.4	37.6	72.8	154.8	194.3	43.8
G(0.5,1)	ARL	49.4	117.1	420.7	910.3	62.7	38.5	82.9	252.4	444.3	46.0
	MRL	35	82	293	623	41	27	58	177	304	31
	SDRL	48.2	114.3	413.2	933.2	65.9	37.3	80.2	245.3	454.7	47.8

because it is a weighted average of squared deviations. For this reason they propose an EWMA chart of the absolute deviations from target ($\mathrm{EWMA_V}$), adjusted for detecting only upward shifts. The statistic of this chart is

$$V_i = \lambda \left| x_i - \mu_0 \right| + (1-\lambda)\max(V_{i-1}, \sigma_0\sqrt{2/\pi}),\ V_0 = \sigma_0\sqrt{2/\pi},$$

where V_0 is the initial value. The above statistic, as in the case of the EWMA$_S$ statistic, is one-sided and can detect only upward shifts. The control limit of this chart is

$$UCL = \sigma_0\sqrt{2/\pi} + h_V\sigma_0\sqrt{1-(2/\pi)}\sqrt{\lambda/\left(2-\lambda\right)},$$

where h_V is a constant specifying the width of the control limit. We have to mention that $\sigma_0\sqrt{2/\pi}$ would be the mean and $\sigma_0\sqrt{1-(2/\pi)}\sqrt{\lambda/\left(2-\lambda\right)}$ would be the asymptotic standard deviation of V_i if the reset was not used. Again, the control limit is not modified and therefore it does not resemble the form of the standard EWMA control limit.

Hawkins and Olwell (1998, p.82) suggested a different statistic for monitoring individual readings for scale changes. Specifically, they recommended the use of the differences $(X_n - \mu_0)$ CUSUMming the square root of their absolute values. In our case, and since we use an EWMA type chart, Maravelakis et al. (2003) introduced such a control chart. Let $H = \sqrt{|x_i - \mu_0|}$, where x_i are our observations. It can be shown that if X is normally distributed $(\mathrm{N}(\mu_0,\, \sigma_0^2))$ then

$$f(h;\sigma_0^2) = \frac{4h}{\sigma_0\sqrt{2\pi}}\exp\left(-\frac{h^4}{2\sigma_0^2}\right), 0 \le h$$

with

$$\begin{aligned} E(h) &= \int_0^\infty \frac{4h^2}{\sigma\sqrt{2\pi}}\exp\left(-\frac{h^4}{2\sigma^2}\right)dh = \frac{2^{3/4}\sigma^{1/2}}{\sqrt{2\pi}}\int_0^\infty \frac{2h^2}{\sigma^{3/2}2^{-1/4}}\exp\left(-\frac{h^4}{2\sigma^2}\right)dh = \\ &= \frac{2^{3/4}\sigma^{1/2}}{\sqrt{2\pi}}\int_0^\infty \left(\frac{h^4}{2\sigma^2}\right)^{-1/4}\exp\left(-\frac{h^4}{2\sigma^2}\right)d\left(\frac{h^4}{2\sigma^2}\right) = \left(2^{3/4}\right)\Gamma\left(3/4\right)\sqrt{\sigma_0/2\pi} \end{aligned}$$

and

$$
\begin{aligned}
E(h^2) &= \int_0^\infty \frac{4h^3}{\sigma\sqrt{2\pi}} \exp\left(-\frac{h^4}{2\sigma^2}\right) dh = \frac{2\sigma^2}{\sigma\sqrt{2\pi}} \int_0^\infty \frac{4h^3}{2\sigma^2} \exp\left(-\frac{h^4}{2\sigma^2}\right) dh = \\
&= \frac{2\sigma^2}{\sigma\sqrt{2\pi}} \int_0^\infty \left(\frac{h^4}{2\sigma^2}\right)^0 \exp\left(-\frac{h^4}{2\sigma^2}\right) d\left(\frac{h^4}{2\sigma^2}\right) = \frac{2\sigma^2\Gamma(1)}{\sigma\sqrt{2\pi}} = \sigma\sqrt{\frac{2}{\pi}}.
\end{aligned}
$$

Table 4.2 (continued) Out-of-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.05$

	Shift	1.6					1.8				
		WR	SR	HO	DP1	DP2	WR	SR	HO	DP1	DP2
N(μ, σ^2)	ARL	34.9	37.7	43.2	38.5	30.8	25.0	27.5	32.2	28.9	22.3
	MRL	27	30	34	30	24	20	22	26	23	18
	SDRL	28.5	28.9	32.8	29.6	25.2	19.5	19.7	22.7	20.7	17.5
G(4,1)	ARL	23.4	32.9	46.6	44.2	22.4	16.8	22.8	31.3	29.4	15.8
	MRL	18	25	35	33	17	13	18	25	23	12
	SDRL	20.5	27.0	37.9	37.1	20.1	14.2	17.7	23.5	22.9	13.6
G(3,1)	ARL	25.4	37.4	55.0	53.2	24.8	18.7	26.4	37.6	35.7	18.0
	MRL	19	28	41	39	18	14	20	29	27	13
	SDRL	22.8	31.8	46.1	46.3	22.8	16.1	21.3	29.6	29.2	15.9
G(2,1)	ARL	27.8	43.7	69.5	69.4	27.7	21.2	31.6	48.1	47.1	20.8
	MRL	20	32	51	50	20	16	24	36	35	15
	SDRL	25.4	38.6	60.7	62.9	26.3	19.0	26.8	40.0	40.7	19.2
G(1,1)	ARL	30.2	54.3	105.7	117.9	32.1	24.4	41.1	75.4	80.1	25.3
	MRL	22	39	76	82	22	18	30	55	57	18
	SDRL	28.6	50.1	97.4	114.1	31.8	22.8	37.2	67.8	75.1	24.6
G(0.5,1)	ARL	31.5	63.0	170.1	260.8	36.2	26.9	50.5	123.9	173.1	29.8
	MRL	22	45	120	178	24	19	36	88	118	20
	SDRL	30.4	60.4	164.2	265.8	37.3	25.6	47.9	117.7	175.3	30.2

Table 4.2 (continued) Out-of-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.1$

Shift		1.2					1.4				
		WR	SR	HO	DP1	DP2	WR	SR	HO	DP1	DP2
$N(\mu, \sigma^2)$	ARL	124.1	123.3	131.8	123.2	113.0	60.7	61.7	68.2	62.2	54.6
	MRL	88	88	94	88	80	44	45	50	45	39
	SDRL	119.8	116.6	123.5	116.2	109.1	56.3	55.6	60.7	55.6	50.5
G(4,1)	ARL	60.5	94.9	147.9	156.7	62.8	34.3	48.7	71.9	73.6	34.2
	MRL	43	67	105	110	44	25	35	52	53	24
	SDRL	58.9	91.0	140.8	152.0	61.8	32.4	45.0	66.2	68.6	32.8
G(3,1)	ARL	58.5	95.4	157.1	171.3	61.8	35.4	52.4	81.1	84.4	35.9
	MRL	41	67	111	120	43	25	37	58	60	25
	SDRL	56.8	91.8	151.1	167.4	61.1	33.7	49.0	75.2	79.7	34.8
G(2,1)	ARL	54.8	94.4	171.7	195.0	59.8	36.0	55.9	94.3	102.0	37.4
	MRL	38	66	120	136	42	25	40	67	72	26
	SDRL	53.5	91.5	165.8	191.4	59.5	34.5	52.7	88.5	98.1	36.6
G(1,1)	ARL	48.3	89.0	199.0	260.6	54.8	35.3	59.6	119.8	146.0	38.5
	MRL	34	62	139	180	38	25	42	85	102	26
	SDRL	47.2	87.0	194.2	259.9	55.1	34.2	57.3	114.8	143.9	38.5
G(0.5,1)	ARL	43.5	81.7	230.3	400.3	51.6	34.6	60.6	151.6	237.8	39.3
	MRL	30	57	161	276	35	24	42	106	164	27
	SDRL	42.8	80.3	226.4	404.6	52.5	33.7	59.4	147.7	240.1	39.9

Then,

$$Var(h) = E(h^2) - [E(h)]^2 = \sigma_0 \left(\sigma \sqrt{\frac{2}{\pi}} - \sqrt{2} \frac{\Gamma^2 (3/4)}{\pi} \right)$$

and the $\mathrm{EWMA_H}$ chart is based on the statistic

$$\begin{aligned} H_i &= \lambda\sqrt{|x_i-\mu_0|}+(1-\lambda)\max\left(H_{i-1},\left(2^{3/4}\right)\Gamma\left(3/4\right)\sqrt{\sigma_0/2\pi}\right), \\ H_0 &= \left(2^{3/4}\right)\Gamma\left(3/4\right)\sqrt{\sigma_0/2\pi} \end{aligned}$$

where H_0 is the initial value. The control limit of this chart is

$$UCL = \left(2^{3/4}\right)\Gamma\left(3/4\right)\sqrt{\sigma_0/2\pi}+h_H\sqrt{\sigma_0\left(\left(2/\sqrt{2\pi}\right)-\sqrt{2}\Gamma^2\left(3/4\right)/\pi\right)\lambda/\left(2-\lambda\right)},$$

where h_H is a constant specifying the width of the control limit. The mean of H_i is $\left(2^{3/4}\right)\Gamma\left(3/4\right)\sqrt{\sigma_0/2\pi}$ and $\sqrt{\sigma_0\left(\left(2/\sqrt{2\pi}\right)-\sqrt{2}\Gamma^2\left(3/4\right)/\pi\right)\lambda/\left(2-\lambda\right)}$ is the asymptotic standard deviation of H_i if the reset is not used. The control limit in this case also is not modified to keep the form of a standard EWMA control limit.

Domangue and Patch (1991) introduced the omnibus EWMA control charts. The statistic used in these charts is $Z_i=(X_i-\mu_0)/\sigma_0$ and the proposed $\mathrm{EWMA_A}$ scheme is

$$A_i=\lambda\left|Z_i\right|^{\alpha}+(1-\lambda)A_{i-1},$$

where the starting value A_0 is set by the practitioner and it is usually equal to the asymptotic mean of A_i. Two different schemes were proposed by Domangue and Patch, one with $a=0.5$ and the second with $a=2$. When we have independent samples from a normal process with mean μ_0 and standard deviation σ_0 Domangue and Patch (1991) showed that the asymptotic mean and variance of A_i for the scheme with $a=1/2$ are $E(A_i)=\left(\sqrt{2}/\pi\right)^{1/2}\Gamma\left(3/4\right)$ and $Var(A_i)=\frac{\sqrt{2}\lambda}{(2-\lambda)\pi}\left[\sqrt{\pi}-\Gamma^2\left(3/4\right)\right]$. In the case of $a=2$ they proved that $E(A_i)=1$ and $Var(A_i)=\frac{2r}{(2-r)}$. Then, the control limit in each case is

$$UCL=E(A_i)+h_AVar(A_i)^{1/2}$$

where h_A is a constant specifying the width of the control limit and either of the schemes signal whenever $A_i \geq UCL$. We have to note here that these schemes can signal only upward because of the way they are constructed. Moreover, as Domangue and Patch indicate these schemes are sensitive to increases in dispersion.

For all the above schemes we observe that they are vulnerable to shifts in the mean apart from the dispersion. Therefore a signal of these charts might be the result of a change in the mean. This deficiency can be resolved by using the moving range (Hawkins and Olwell (1998, p.82)) or by calculating at each point in time (observation) an estimate of the mean (MacGregor and Harris (1993)). However, the use of either of these techniques might lead to other problems such as dependence of the observations and since they involve cumbersome calculations, they are not considered here.

4.3.3 Methods of evaluating control charts performance and their computation

In the context of EWMA charts there are three ways of computing the previously stated measures of performance. The integral equation method, the Markov chain method and a simulation study (see e.g., Brook and Evans (1972), Lucas and Saccucci (1990) and Domangue and Patch (1991)). The integral equation method is an accurate method but it can not be computed in all cases. The Markov chain method can be implemented in the cases that the former method can not, but we need to discretise the continuity of the process using many steps. The simulation study is easy in the implementation and, when using a large number of iterations, the results are very accurate. In the following calculations simulation is used and we repeat the simulation 200001 times for each entry in the tables.

In order to study the effect of non-normality in the performance of the EWMA charts for dispersion we used the same types of distributions as in Borror et al. (1999) and Stoumbos and Reynolds (2000); symmetric and skewed ones. Specifically, we simulated observations in the skewed case from the Gamma(a, b) distribution with probability density function

$$f(x;\alpha,b) = \left\{ \begin{array}{cc} \frac{b^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} \exp(-bx) & x > 0 \\ 0, & x \leq 0 \end{array} \right\},$$

Table 4.2 (continued) Out-of-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.1$

Shift		1.6					1.8				
		WR	SR	HO	DP1	DP2	WR	SR	HO	DP1	DP2
$N(\mu,\sigma^2)$	ARL	37.3	38.5	43.4	39.7	33.6	26.1	27.4	31.4	28.8	23.6
	MRL	27	29	32	30	25	20	21	24	22	18
	SDRL	33.4	32.9	36.6	33.4	29.9	22.5	22.2	25.2	22.9	20.2
G(4,1)	ARL	22.4	29.8	42.1	41.9	21.9	16.1	20.6	28.2	27.5	15.5
	MRL	16	22	31	31	16	12	16	21	21	11
	SDRL	20.5	26.3	36.9	37.2	20.3	14.3	17.4	23.3	23.2	14.0
G(3,1)	ARL	24.1	33.4	49.4	49.9	24.0	17.9	23.6	33.6	33.3	17.4
	MRL	17	24	36	36	17	13	18	25	25	13
	SDRL	22.4	30.1	44.0	45.4	22.7	16.3	20.6	28.6	29.0	16.0
G(2,1)	ARL	25.9	37.9	59.8	62.7	26.4	19.9	27.7	42.1	42.9	19.9
	MRL	18	27	43	45	19	14	20	31	31	14
	SDRL	24.4	34.9	54.6	58.7	25.4	18.6	25.0	37.3	39.0	18.7
G(1,1)	ARL	27.6	43.7	81.3	94.0	29.2	22.6	34.0	59.9	66.6	23.4
	MRL	20	31	58	66	20	16	24	43	47	16
	SDRL	26.4	41.4	77.1	91.6	28.9	21.4	31.7	55.5	63.6	22.8
G(0.5,1)	ARL	28.6	47.6	108.9	157.4	31.9	24.4	39.3	83.7	113.1	26.8
	MRL	20	34	76	108	22	17	28	59	78	18
	SDRL	27.7	46.1	105.6	158.4	32.3	23.5	37.7	80.7	112.9	26.8

where the mean is α/b and the variance is α/b^2. In the remaining of the chapter we set b equal to unity without loss of generality. Under this condition as α increases the gamma distribution approaches the normal. In the symmetric case we simulated observations from the t(k) distribution with probability density function

$$f(x;k) = \frac{\Gamma\left(\left(k+1\right)/2\right)}{\sqrt{\pi}\Gamma\left(k/2\right)} \frac{1}{\left(\left(x^2/k\right)+1\right)^{(k+1)/2}}, -\infty < x < \infty,$$

Table 4.2 (continued) Out-of-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.2$

Shift		1.2					1.4				
		WR	SR	HO	DP1	DP2	WR	SR	HO	DP1	DP2
N(μ, σ^2)	ARL	136.7	133.4	137.7	132.0	128.8	69.1	67.0	71.1	67.6	63.5
	MRL	95	94	97	93	90	49	48	51	48	45
	SDRL	134.5	129.8	133.1	128.0	127.1	67.1	63.6	66.6	63.4	61.4
G(4,1)	ARL	56.0	76.9	116.3	128.7	57.6	33.1	42.1	59.8	63.8	32.9
	MRL	39	54	82	90	40	23	30	42	45	23
	SDRL	54.9	74.7	113.1	126.5	56.8	32.0	40.1	56.7	61.2	32.0
G(3,1)	ARL	53.5	75.2	118.5	133.7	55.6	33.5	43.9	65.0	70.7	34.0
	MRL	37	53	83	93	39	24	31	46	50	24
	SDRL	52.8	73.5	115.3	131.4	55.2	32.3	42.2	62.0	68.4	33.4
G(2,1)	ARL	49.6	71.4	119.6	140.2	52.3	33.5	45.4	70.2	79.4	34.5
	MRL	35	50	84	97	36	24	32	49	56	24
	SDRL	48.7	69.9	116.4	138.7	51.9	32.6	44.0	67.5	77.1	34.0
G(1,1)	ARL	43.6	64.0	118.1	151.0	46.8	32.5	45.4	77.7	94.6	34.3
	MRL	31	45	83	105	33	23	32	55	66	24
	SDRL	42.7	62.6	115.8	150.3	46.8	31.6	44.1	75.3	93.6	33.9
G(0.5,1)	ARL	39.5	57.8	117.1	174.9	43.4	31.5	44.6	83.7	118.3	34.3
	MRL	28	40	81	121	30	22	31	59	82	24
	SDRL	38.8	57.2	115.6	175.8	43.4	30.9	43.6	82.1	118.5	34.2

where k are the degrees of freedom, the mean is 0 and the variance is $k/(k-2)$. The t distribution is symmetric about 0 but it has more probability in the tails than the normal. Moreover, as the degrees of freedom increase, the t distribution approaches the normal.

In the simulation algorithm, the parameter values we simulated from, are α=0.5, 1, 2, 3, 4 and b=1 in the gamma case, and k= 4, 6, 8, 10, 20, 30, 40, 50 in the t distribution case. The steps of the algorithm are the following

Step 1. Set the values of μ_0 and σ_0

Step 2. Set the values of λ and the constants specifying the width of the control limits (h_S, h_V, h_H, h_A) and calculate the control limits.

Step 3. Generate a value from gamma(α,1) [from a t(k) distribution] and calculate the appropriate statistic in each case.

Step 4. Repeat Step 3 until the statistic computed crosses the upper control limit and record the sample this happens.

Step 5. Repeat Steps 3 to 4 200001 times.

Step 6. Obtain estimates of the ARL and SDRL values.

Step 7. Sort the 200001 values and set observation 100001 equal to the MRL.

Evidently, the above algorithm is used for calculating the in-control ARL, MRL and SDRL values. For the out-of-control cases Step 3 is properly modified. In Step 1, the in-control mean when we are in the gamma case is equal to α/b and the variance is α/b^2. When we have a t distribution the in-control mean is 0 and the variance is $k/(k-2)$. The values under the normal distribution are calculated also in each case for studying the non-normality effect. The values of λ chosen are 0.05, 0.1 and 0.2 which are the usually chosen values for studying the non-normality effect (see e.g., Borror et al. (1999), Stoumbos and Reynolds (2000), Reynolds and Stoumbos (2001)). The values of (h_S, h_V, h_H, h_A) are chosen in a way that under normality they give the same in-control value for ARL approximately 370.4. Also, in all the cases, results are displayed for asymptotic control limits. Finally, all the out-of-control computations performed in this chapter are made under the assumption of immediate occurrence of the shift at the beginning of the process.

Table 4.2 (continued) Out-of-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.2$

Shift		1.6					1.8				
		WR	SR	HO	DP1	DP2	WR	SR	HO	DP1	DP2
N(μ, σ^2)	ARL	42.0	41.4	44.9	42.2	38.6	29.1	28.8	31.6	29.9	26.8
	MRL	30	30	32	31	28	21	21	23	22	19
	SDRL	40.0	38.1	40.9	38.3	36.7	27.1	25.7	27.7	26.2	24.7
G(4,1)	ARL	22.0	26.6	36.1	37.5	21.6	15.8	18.6	24.3	24.9	15.5
	MRL	16	19	26	27	15	11	14	18	18	11
	SDRL	20.8	24.6	33.1	34.8	20.6	14.7	16.8	21.5	22.5	14.5
G(3,1)	ARL	23.2	29.0	40.7	43.2	23.2	17.3	20.9	28.4	29.5	17.1
	MRL	16	21	29	31	16	12	15	21	21	12
	SDRL	22.1	27.3	37.9	40.9	22.3	16.3	19.2	25.7	27.1	16.2
G(2,1)	ARL	24.5	31.6	47.0	51.4	24.9	19.1	23.9	33.8	36.2	19.1
	MRL	17	22	33	36	17	14	17	24	26	14
	SDRL	23.6	30.0	44.0	49.3	24.2	18.1	22.3	31.2	34.0	18.3
G(1,1)	ARL	25.7	34.6	55.8	66.0	26.6	21.1	27.7	43.0	49.2	21.6
	MRL	18	24	39	46	19	15	20	30	35	15
	SDRL	24.9	33.5	53.3	64.8	26.2	20.4	26.5	40.7	47.5	21.1
G(0.5,1)	ARL	26.4	36.2	64.6	87.2	28.1	22.6	30.6	52.0	67.6	24.1
	MRL	19	25	45	61	20	16	21	37	47	17
	SDRL	25.7	35.3	63.1	86.6	27.8	21.9	29.7	50.5	67.4	23.8

4.3.4 Results

In Tables 4.1 through 4.4, we have the results of the EWMA charts for the dispersion for the five different charts ($EWMA_S$, $EWMA_V$, $EWMA_H$ and $EWMA_A$ for α=1/2 and α=2). We have results for three combinations of λ and the corresponding h_S, h_V, h_H

and h_A values. In the second row of table 4.1 we have the five different h_S, h_V, h_H and h_A values, which are calculated so as to give under normality an in-control value of ARL equal to 370.4. The same values for these h constants are used in Tables 4.2, 4.3 and 4.4 and for this reason they are not displayed. The third column (WR) in each Table is the ARL, MRL and SDRL values for the $\mathrm{EWMA_S}$, the fourth column (SR) is for the $\mathrm{EWMA_V}$, the fifth column (HO) is for the $\mathrm{EWMA_H}$ and the sixth (DP1) and seventh (DP2) columns are for the $\mathrm{EWMA_A}$ with α=0.5 and α=2 respectively.

In Table 4.1, the results for the in-control case for the gamma distribution are displayed and in Table 4.3 the corresponding ones for the t distribution (ARL(0)). In Table 4.2, we have the results in the out-of-control case for the Gamma distribution and in Table 4.4 the corresponding ones for the t distribution (ARL(1)). In each Table we have computed additionally the ARL, MRL and SDRL values for the normal distribution to identify the non normality effect. The shift in the out-of-control cases is in the in-control process variance, whose value is set at the first Step of the algorithm, by multiplying it with 1.2, 1.4, 1.6 and 1.8.

The conclusions drawn from these tables are the following. When the process is in-control, the $\mathrm{EWMA_H}$ chart for $\lambda = 0.1$ has a satisfactory non normality performance. Additionally, the $\mathrm{EWMA_A}$ chart when $a = 0.5$ for $\lambda = 0.2$ gives also results comparable to the normal ones when we are in-control. One also concludes that the other charts are much less efficient regarding non normality for every combination of the smoothing parameter and the process parameters presented. Most of the times they lead to a larger number of false alarms than the nominal. However, the $\mathrm{EWMA_H}$ and $\mathrm{EWMA_A}$ for $a = 0.5$ can give for certain parameters, very large ARL values. As the value of α in the gamma case and k in the t-distribution case, become larger so does the ARL and MRL for $\mathrm{EWMA_S}$, $\mathrm{EWMA_V}$ and $\mathrm{EWMA_A}$ for $a = 2$. On the other hand, the ARL and MRL values for the $\mathrm{EWMA_H}$ and $\mathrm{EWMA_A}$ for $\alpha = 0.5$ decrease when $\lambda = 0.05$, $\lambda = 0.1$ and increase for $\lambda = 0.2$.

In the out-of-control cases, as the shift increases the non normality effect decreases.

Table 4.3. In-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.05$

		WR	SR	HO	DP1	DP2
$N(\mu, \sigma^2)$	ARL	370.4	370.4	370.4	370.4	370.4
	MRL	260	260	264	259	257
	SDRL	361.3	358.1	353.6	361.8	368.3
t_4	ARL	112.6	271.0	792.9	2208	147.4
	MRL	79	189	549	1515	100
	SDRL	110.8	267.4	787.3	2251	151.4
t_6	ARL	138.6	297.8	585.2	946.5	170.9
	MRL	97	209	410	653	117
	SDRL	135.8	290.6	573.1	953.5	173.4
t_8	ARL	165.6	318.3	589.9	695.3	195.3
	MRL	117	224	412	481	134
	SDRL	161.4	310.5	580.7	695.6	197.8
t_{10}	ARL	186.0	329.9	476.8	591.8	216.2
	MRL	131	231	336	411	149
	SDRL	180.9	321.1	462.4	588.2	218.0
t_{20}	ARL	252.3	352.6	416.0	456.7	276.4
	MRL	178	249	292	318	192
	SDRL	244.8	341.9	402.2	452.8	274.8
t_{30}	ARL	285.8	358.8	401.2	424.1	303.1
	MRL	200	252	282	296	211
	SDRL	279.8	346.4	389.3	417.6	301.4
t_{40}	ARL	302.5	361.5	390.6	409.7	319.5
	MRL	213	254	275	285	222
	SDRL	293.3	349.5	377.5	404.3	317.4
t_{50}	ARL	314.9	366.4	387.7	400.8	327.6
	MRL	221	256	272	279	227
	SDRL	307.0	356.5	374.1	395.1	324.8

Table 4.3 (continued) In-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.1$

		WR	SR	HO	DP1	DP2
$\mathrm{N}(\mu, \sigma^2)$	ARL	370.4	370.4	370.4	370.4	370.4
	MRL	259	257	260	259	258
	SDRL	365.9	360.8	359.2	363.6	367.4
t_4	ARL	97.7	187.4	441.5	882.4	116.4
	MRL	68	131	307	609	80
	SDRL	96.1	185.5	438.1	890.9	117.5
t_6	ARL	120.9	219.9	409.6	590.9	140.1
	MRL	84	153	288	410	97
	SDRL	119.6	217.4	399.3	590.8	140.2
t_8	ARL	145.2	247.1	400.9	508.8	163.9
	MRL	101	173	279	354	114
	SDRL	143.0	243.0	395.4	506.8	164.1
t_{10}	ARL	167.7	269.7	394.1	470.4	185.0
	MRL	117	189	275	326	128
	SDRL	165.0	264.5	388.0	467.3	184.9
t_{20}	ARL	233.3	316.5	380.6	412.9	250.2
	MRL	163	222	267	287	173
	SDRL	230.0	310.3	372.7	408.4	249.3
t_{30}	ARL	270.6	334.3	378.3	397.1	283.2
	MRL	190	234	264	277	196
	SDRL	264.6	326.7	371.1	392.2	283.8
t_{40}	ARL	291.7	341.8	375.0	390.1	301.0
	MRL	205	239	262	272	210
	SDRL	286.2	336.0	367.5	385.8	298.9
t_{50}	ARL	305.1	348.1	373.9	386.9	314.3
	MRL	213	243	263	269	218
	SDRL	301.8	341.1	363.4	381.4	312.6

Table 4.3 (continued) In-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.2$

		WR	SR	HO	DP1	DP2
$N(\mu, \sigma^2)$	ARL	370.4	370.4	370.4	370.4	370.4
	MRL	256	257	257	259	258
	SDRL	368.9	363.4	363.3	366.4	368.8
t_4	ARL	86.7	130.1	238.3	383.0	96.0
	MRL	60	91	166	266	67
	SDRL	86.4	128.7	235.7	383.1	95.8
t_6	ARL	109.2	159.2	265.3	353.8	118.0
	MRL	76	111	186	245	82
	SDRL	108.0	158.1	261.3	354.3	117.5
t_8	ARL	131.2	187.8	283.9	349.7	140.9
	MRL	91	131	198	243	98
	SDRL	130.8	186.1	278.8	347.4	140.6
t_{10}	ARL	152.1	212.3	302.6	353.4	162.0
	MRL	106	148	211	246	112
	SDRL	150.3	210.0	298.0	352.3	162.5
t_{20}	ARL	220.0	275.9	332.2	361.4	229.3
	MRL	154	192	232	251	159
	SDRL	217.8	273.1	328.0	357.4	228.7
t_{30}	ARL	257.1	303.7	347.4	365.5	264.7
	MRL	178	212	243	255	184
	SDRL	255.4	299.4	343.2	362.4	262.3
t_{40}	ARL	279.3	321.1	351.7	365.5	285.6
	MRL	195	223	245	255	198
	SDRL	276.2	318.7	346.0	362.8	284.0
t_{50}	ARL	294.7	327.3	355.1	366.1	298.9
	MRL	205	228	247	254	207
	SDRL	292.6	323.0	351.1	362.4	298.0

Table 4.4. Out–of-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.05$

	Shift	1.2					1.4				
		WR	SR	HO	DP1	DP2	WR	SR	HO	DP1	DP2
$N(\mu, \sigma^2)$	ARL	113.3	116.2	126.1	114.4	100.8	55.6	58.4	65.9	58.6	48.8
	MRL	81	84	92	82	72	41	44	50	44	37
	SDRL	105.5	105.1	113.1	104.5	94.2	48.2	48.5	54.2	48.7	42.5
t_4	ARL	75.5	159.5	417.6	930.3	91.0	59.5	116.2	283.6	556.6	68.4
	MRL	53	112	291	638	62	42	82	199	381	46
	SDRL	73.6	155.8	410.6	946.7	92.9	57.7	113.1	277.1	569.0	69.6
t_6	ARL	74.9	140.2	275.0	386.2	83.4	53.3	93.2	177.0	231.1	57.1
	MRL	53	99	193	267	57	38	66	126	160	39
	SDRL	72.0	134.5	265.4	386.8	83.8	50.9	88.0	168.2	230.8	56.5
t_8	ARL	77.1	134.6	232.9	284.5	82.2	52.1	85.2	147.5	170.7	54.0
	MRL	55	96	165	198	57	37	61	105	119	38
	SDRL	73.8	127.8	223.4	281.8	81.8	49.4	79.2	138.0	167.2	52.6
t_{10}	ARL	78.7	131.2	212.2	244.2	82.5	51.6	81.4	133.4	147.1	52.4
	MRL	56	93	151	170	57	37	59	96	103	37
	SDRL	75.2	124.4	201.6	240.1	81.3	48.5	75.3	123.4	141.9	50.9
t_{20}	ARL	83.6	126.2	180.6	187.6	84.6	50.7	75.3	112.3	114.9	50.3
	MRL	60	90	129	132	59	37	55	81	82	36
	SDRL	79.1	118.1	169.5	180.7	82.1	47.0	68.1	102.0	108.1	47.5
t_{30}	ARL	86.1	125.1	171.6	175.4	85.8	50.7	73.6	106.6	107.1	49.6
	MRL	61	90	123	123	60	37	54	77	77	35
	SDRL	81.2	116.9	159.2	168.4	83.2	46.7	66.1	96.1	99.9	47.0
t_{40}	ARL	87.1	124.9	168.0	169.1	86.1	50.5	72.8	103.9	104.0	49.5
	MRL	62	89	120	120	61	37	53	76	74	36
	SDRL	82.2	116.2	156.1	161.0	83.2	46.8	65.0	93.5	96.8	46.7
t_{50}	ARL	87.7	124.1	165.7	165.6	86.5	50.7	72.1	102.4	101.9	49.2
	MRL	63	89	119	117	61	37	53	75	73	35
	SDRL	82.5	115.3	153.6	157.5	83.2	46.6	64.6	91.8	94.5	46.0

Table 4.4 (continued) Out–of-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.05$

	Shift			1.6					1.8		
		WR	SR	HO	DP1	DP2	WR	SR	HO	DP1	DP2
$N(\mu, \sigma^2)$	ARL	34.9	37.7	43.2	38.5	30.8	25.0	27.5	32.2	28.9	22.3
	MRL	27	30	34	30	24	20	22	26	23	18
	SDRL	28.5	28.9	32.8	29.6	25.2	19.5	19.7	22.7	20.7	17.5
t_4	ARL	50.7	95.1	220.0	397.5	57.1	45.6	81.9	183.3	311.6	50.1
	MRL	36	67	155	273	39	32	58	129	213	34
	SDRL	48.8	91.5	213.2	406.4	57.6	43.8	78.8	176.8	318.2	50.4
t_6	ARL	43.3	72.1	133.6	166.5	45.3	37.3	60.5	110.0	132.8	38.6
	MRL	31	51	95	116	31	27	44	79	93	27
	SDRL	41.0	67.7	125.3	164.5	44.6	35.1	56.1	102.2	129.8	37.6
t_8	ARL	40.9	65.0	109.9	124.2	41.8	34.8	53.5	90.0	99.8	35.1
	MRL	29	47	79	87	29	25	39	65	70	25
	SDRL	38.3	59.8	101.2	120.4	40.3	32.3	48.5	81.9	95.4	33.6
t_{10}	ARL	39.7	61.2	99.0	107.5	39.7	33.6	50.3	81.0	86.7	33.3
	MRL	29	44	71	76	28	25	37	59	62	24
	SDRL	37.0	55.8	90.1	102.0	37.9	31.0	45.2	72.9	81.4	31.5
t_{20}	ARL	37.9	55.3	83.6	84.2	37.2	31.3	44.8	67.9	68.4	30.7
	MRL	28	41	61	60	27	23	33	50	49	22
	SDRL	34.8	49.0	74.3	78.0	34.8	28.5	39.2	59.4	62.1	28.5
t_{30}	ARL	37.5	53.4	79.0	79.0	36.4	30.8	43.7	64.3	63.9	30.1
	MRL	27	39	58	57	26	23	32	48	47	22
	SDRL	34.1	47.2	69.4	72.2	33.8	27.7	37.9	55.4	57.2	27.8
t_{40}	ARL	37.2	52.7	77.1	76.2	36.2	30.5	43.0	62.9	61.8	29.5
	MRL	27	39	57	55	26	22	32	47	45	22
	SDRL	33.8	46.1	67.5	69.1	33.5	27.5	37.2	54.1	55.0	27.1
t_{50}	ARL	37.2	52.4	76.2	75.0	36.1	30.4	42.6	61.9	60.8	29.4
	MRL	27	39	56	55	26	22	32	46	45	21
	SDRL	33.7	45.8	66.6	67.6	33.3	27.3	36.8	52.9	54.1	26.9

Table 4.4 (continued) Out–of-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.1$

	Shift	1.2					1.4				
		WR	SR	HO	DP1	DP2	WR	SR	HO	DP1	DP2
$N(\mu, \sigma^2)$	ARL	124.1	123.3	131.8	123.2	113.0	60.7	61.7	68.2	62.2	54.6
	MRL	88	88	94	88	80	44	45	50	45	39
	SDRL	119.8	116.6	123.5	116.2	109.1	56.3	55.6	60.7	55.6	50.5
t_4	ARL	67.4	116.8	254.6	449.8	76.3	53.8	88.6	182.3	300.3	59.6
	MRL	47	82	178	310	53	38	62	127	207	41
	SDRL	66.0	114.7	250.4	454.0	76.7	52.5	86.9	179.2	302.4	59.6
t_6	ARL	68.1	110.8	202.3	270.4	73.4	49.3	76.0	134.3	171.4	52.3
	MRL	48	78	142	187	51	35	54	95	119	36
	SDRL	66.3	107.6	197.1	269.9	73.5	48.0	73.0	129.5	170.6	51.7
t_8	ARL	70.7	110.2	184.7	224.3	75.2	48.5	72.1	118.7	139.7	50.4
	MRL	50	78	130	156	52	34	51	84	98	35
	SDRL	69.1	106.5	178.6	222.0	74.2	46.7	68.8	113.3	136.8	49.4
t_{10}	ARL	73.0	111.1	175.9	203.6	76.4	48.5	70.1	111.2	126.4	49.5
	MRL	51	79	124	142	53	34	50	79	89	35
	SDRL	70.9	107.2	169.2	200.5	75.6	46.7	66.6	105.4	122.7	48.3
t_{20}	ARL	79.7	112.9	160.8	172.5	81.4	48.4	67.2	99.1	105.4	48.8
	MRL	56	80	113	121	57	34	48	71	75	34
	SDRL	77.2	107.9	154.0	167.5	80.0	46.1	63.1	92.9	101.1	47.0
t_{30}	ARL	82.9	114.0	156.5	164.2	83.5	48.7	66.6	95.6	100.3	48.6
	MRL	58	81	111	116	59	35	48	68	71	34
	SDRL	80.5	109.1	149.4	159.0	81.4	46.1	62.4	89.3	95.7	47.0
t_{40}	ARL	84.3	114.9	154.2	160.8	85.0	49.2	66.0	93.7	97.3	48.5
	MRL	60	81	109	113	60	35	47	67	69	34
	SDRL	81.2	109.9	147.1	155.3	83.0	47.0	61.9	87.3	92.6	46.9
t_{50}	ARL	85.3	115.4	153.2	158.8	85.4	49.2	66.0	93.2	96.6	48.6
	MRL	60	82	109	112	60	35	47	67	68	34
	SDRL	82.6	110.4	145.2	153.8	83.7	47.1	61.9	87.0	91.7	46.8

Table 4.4 (continued) Out–of-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.1$

	Shift	1.6					1.8				
		WR	SR	HO	DP1	DP2	WR	SR	HO	DP1	DP2
$N(\mu, \sigma^2)$	ARL	37.3	38.5	43.4	39.7	33.6	26.1	27.4	31.4	28.8	23.6
	MRL	27	29	32	30	25	20	21	24	22	18
	SDRL	33.4	32.9	36.6	33.4	29.9	22.5	22.2	25.2	22.9	20.2
t_4	ARL	46.3	73.8	145.8	229.1	50.4	41.6	64.8	124.2	189.9	45.1
	MRL	33	52	102	158	35	29	46	87	131	31
	SDRL	45.3	71.7	142.4	231.2	50.5	40.4	62.8	120.9	191.9	45.0
t_6	ARL	40.4	59.9	102.9	127.7	42.0	35.1	51.0	85.5	103.8	36.2
	MRL	28	42	73	89	29	25	36	61	72	25
	SDRL	39.0	57.7	98.3	126.2	41.2	33.7	48.6	80.8	101.7	35.4
t_8	ARL	38.4	55.2	89.6	103.4	39.3	32.8	46.4	73.4	84.0	33.4
	MRL	27	39	64	73	28	23	33	52	59	24
	SDRL	37.0	52.3	84.4	100.2	38.4	31.3	43.8	68.8	80.7	32.3
t_{10}	ARL	37.6	52.8	82.9	92.8	38.3	31.8	44.0	67.6	75.0	32.0
	MRL	27	38	59	65	27	23	32	49	53	23
	SDRL	35.9	49.7	77.8	89.1	37.0	30.3	41.1	62.7	71.5	30.9
t_{20}	ARL	36.5	49.2	72.7	77.0	36.1	30.0	40.4	59.0	62.2	29.8
	MRL	26	35	52	55	26	21	29	43	44	21
	SDRL	34.4	45.5	67.0	72.7	34.6	28.3	37.1	53.9	58.2	28.4
t_{30}	ARL	36.2	48.5	70.2	73.1	35.9	29.8	39.2	56.8	59.2	29.4
	MRL	26	35	51	52	25	21	28	41	42	21
	SDRL	34.3	45.0	64.7	68.5	34.2	27.8	36.0	51.5	55.1	27.9
t_{40}	ARL	36.2	47.9	68.9	71.4	35.5	29.5	38.9	55.7	57.3	29.1
	MRL	26	35	50	51	25	21	28	40	41	21
	SDRL	34.2	44.2	63.0	67.0	33.9	27.6	35.8	50.3	53.2	27.5
t_{50}	ARL	35.9	47.7	68.3	70.4	35.5	29.4	38.7	55.0	56.5	28.8
	MRL	26	34	49	50	25	21	28	40	41	21
	SDRL	33.9	44.0	62.4	65.9	33.8	27.5	35.3	49.6	52.4	27.4

Table 4.4 (continued) Out–of-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.2$

	Shift	1.2					1.4				
		WR	SR	HO	DP1	DP2	WR	SR	HO	DP1	DP2
$N(\mu, \sigma^2)$	ARL	136.7	133.4	137.7	132.0	128.8	69.1	67.0	71.1	67.6	63.5
	MRL	95	94	97	93	90	49	48	51	48	45
	SDRL	134.5	129.8	133.1	128.0	127.1	67.1	63.6	66.6	63.4	61.4
t_4	ARL	61.2	87.0	149.0	223.9	65.8	49.4	68.1	112.3	162.1	52.7
	MRL	43	61	104	155	46	35	48	79	113	37
	SDRL	60.3	85.8	146.8	224.1	65.6	48.6	66.7	110.2	161.8	52.4
t_6	ARL	62.7	86.3	136.7	175.8	66.0	46.2	60.9	94.3	117.5	47.9
	MRL	44	60	96	122	46	32	43	66	82	33
	SDRL	61.8	84.7	133.9	174.8	65.5	45.4	59.5	91.5	116.4	47.4
t_8	ARL	66.1	88.8	135.1	161.9	68.8	45.9	59.5	88.3	104.7	47.3
	MRL	46	62	95	113	48	32	42	62	73	33
	SDRL	64.9	87.2	131.3	160.1	68.1	45.0	57.8	85.5	102.9	46.6
t_{10}	ARL	68.8	91.0	133.8	156.3	71.5	46.2	59.5	85.7	98.4	47.1
	MRL	48	64	94	109	50	32	42	61	69	33
	SDRL	67.5	89.0	130.2	154.6	70.5	45.1	57.7	82.3	96.4	46.2
t_{20}	ARL	76.9	98.1	133.6	147.4	78.0	47.2	58.9	81.5	88.9	47.5
	MRL	54	69	94	103	54	33	42	58	63	33
	SDRL	75.4	95.6	129.8	144.9	77.1	46.0	56.7	78.0	86.2	46.6
t_{30}	ARL	80.6	100.7	134.1	144.6	81.0	47.9	59.0	80.1	86.6	47.9
	MRL	56	70	94	101	56	34	41	57	61	34
	SDRL	79.2	98.7	130.4	141.2	80.0	46.7	56.8	76.8	84.3	46.9
t_{40}	ARL	82.4	102.2	133.9	143.1	82.6	48.0	59.0	79.4	85.7	48.2
	MRL	57	72	94	100	58	34	42	56	60	34
	SDRL	81.0	99.4	129.7	139.8	81.6	46.9	56.9	76.3	83.2	47.2
t_{50}	ARL	83.4	103.5	133.9	143.3	83.9	48.4	58.9	79.3	85.3	47.9
	MRL	58	73	94	100	58	34	42	56	60	34
	SDRL	82.3	100.6	129.6	140.2	83.1	47.1	56.5	75.8	82.9	46.9

Table 4.4 (continued) Out–of-control ARL, MRL and SDRL values for upward shifts $\lambda = 0.2$

	Shift	1.6					1.8				
		WR	SR	HO	DP1	DP2	WR	SR	HO	DP1	DP2
$N(\mu, \sigma^2)$	ARL	42.0	41.4	44.9	42.2	38.6	29.1	28.8	31.6	29.9	26.8
	MRL	30	31	32	31	28	21	21	23	22	19
	SDRL	40.0	38.1	40.9	38.3	36.7	27.1	25.7	27.7	26.2	24.7
t_4	ARL	42.6	57.3	92.6	130.4	45.1	38.6	51.2	80.4	111.2	40.5
	MRL	30	40	65	91	31	27	36	56	77	28
	SDRL	42.0	56.2	90.4	130.2	44.9	37.8	50.1	78.4	111.4	40.1
t_6	ARL	38.0	49.2	73.7	90.4	39.1	33.2	42.3	62.3	75.5	33.9
	MRL	27	35	52	63	27	23	30	44	53	24
	SDRL	37.1	47.9	71.4	89.0	38.5	32.3	40.8	59.9	74.3	33.4
t_8	ARL	36.7	46.4	67.3	79.1	37.4	31.5	39.3	55.9	65.1	31.8
	MRL	26	33	47	55	26	22	28	40	46	22
	SDRL	35.7	45.0	64.9	77.3	36.5	30.5	37.9	53.5	63.4	31.1
t_{10}	ARL	36.2	45.3	64.6	73.6	36.7	30.5	38.1	53.3	60.3	30.8
	MRL	26	32	46	52	26	22	27	38	42	22
	SDRL	35.1	43.6	61.8	71.8	36.0	29.4	36.5	50.5	58.5	30.1
t_{20}	ARL	35.5	43.4	59.7	65.6	35.6	29.3	35.5	48.4	52.7	29.4
	MRL	25	31	43	46	25	21	25	34	37	21
	SDRL	34.3	41.6	56.5	63.6	34.7	28.2	33.8	45.8	50.5	28.4
t_{30}	ARL	35.4	43.0	58.3	63.0	35.2	29.1	35.0	46.9	50.9	29.0
	MRL	25	30	41	45	25	21	25	34	36	20
	SDRL	34.3	41.0	55.2	60.6	34.3	28.0	33.3	44.0	48.8	28.0
t_{40}	ARL	35.3	42.5	57.9	62.1	35.1	29.0	34.6	46.5	50.0	28.7
	MRL	25	30	41	44	25	21	25	33	36	20
	SDRL	34.0	40.3	54.6	59.8	34.2	27.8	32.8	43.5	47.5	27.7
t_{50}	ARL	35.3	42.6	57.2	61.3	35.1	28.7	34.6	46.1	49.4	28.6
	MRL	25	30	41	43	25	20	25	33	35	20
	SDRL	34.2	40.5	54.1	58.6	34.1	27.6	32.7	43.2	47.2	27.7

Note that a direct comparison of the different schemes is not possible because they do not have the same in-control ARL or MRL values. We observe that the out-of-control ARL performance of the charts that have a good in-control one, is far from the normal. They do not give that fast an out-of-control signal, therefore they lose the ability of the EWMA charts to identify an out-of-control situation for small shifts quickly.

Consequently, the $\mathrm{EWMA_H}$ and $\mathrm{EWMA_A}$ (for α=0.5) charts are a very good choice when normality is questionable for specific values of the smoothing parameter λ when our process is in-control. In the out-of-control cases they give disappointing results. The $\mathrm{EWMA_S}$ and $\mathrm{EWMA_A}$ for α=2 charts are not recommended since their performance in both in-control and out-of-control situations is far from the normal. The $\mathrm{EWMA_V}$ chart does not perform well for skewed distributions but in the symmetric case the results are better for small values of λ. Generally we can say that none of the presented schemes is robust to the normality assumption.

4.3.5 Discussion

The research for the non-normality effect of the EWMA control charts for process dispersion was conducted in a way for the results drawn, to hold for data coming from any distribution without a need to know this distribution. However, in the particular case that we know explicitly the distribution our data are coming from, we may use a transformation of the data to the normal. Such a transformation is given in Hawkins and Olwell (p. 163, 1998) and it has been used also by Quesenberry (1995a) and Chen et al. (2001) in the context of EWMA charts. This transformation not only achieves approximate normality but also independence of the resulting data.